Math 360 – Abstract Algebra – Assignment 1

1. For each of the following, decide whether the given sets with the given multiplication forms a group. If it does for a group, do the following: explain why it is closed under the operation, state the identity element, and describe how you would get the inverse of an arbitrary element, not a particular element (for instance, if I gave you $(\mathbf{Z}, +)$, you wouldn't say that the inverse of 5 is -5, you would say that the inverse of a is $-a$). If it does not form a group, demonstrate that it fails at least one of the axioms. property which

a) $(\mathbf{Z}, \circ)$ where $a \circ b = a + b - 37$.

b) $(\mathbf{R}, \circ)$ where $x \circ y = x + y - xy$.

c) $(\mathbf{R}^\times, \times)$, where $\mathbf{R}^\times$ means all the non-zero real numbers.

d) $(SL_2(\mathbf{R}), \circ)$, where $SL_2(\mathbf{R})$ are all the 2 by 2 inverible matrices with real entries, and $\circ$ means matrix multiplication.

2. Do exercise E from chapter 2.

3. a) We have seen several solutions to the two-nail picture-hanging problem. One of them is $ab^{-1}a^{-1}b$. List thee other solutions.

b) Find a solution to the three-nail picture-hanging problem. Remember, the picture must fall if any one of the three nails is removed.

c) (Bonus) Find a solution to the four-nail picture-hanging problem. Find a solution to the n-nail picture-hanging problem.