Math 360 – Abstract Algebra – Assignment 2

1. a) Complete the multiplication table for D_4 that we began in class.

b) List all the elements of D_3 (give me a short explaination of each e.g. This is a rotation of this many degrees clockwise, or this is a reflection over blah...blah...blah, pictures are fine if you want to save words). Write the multiplication table for this group.

2. Tell me the order of each element of D_3.

3. Let G be a group and assume that $a, b \in G$ commute (that is $ab = ba$, even though G may not be an abelian group).

a) Prove that a^{-1} and b^{-1} commute.

You don't actually have to do part a) because I'm going to show you one possible solution right now (though there are many others):

Assume $ab = ba$

$$\begin{aligned} a^{-1}b^{-1} &= (ba)^{-1} && \text{by proposition on inverses} \\ &= (ab)^{-1} && \text{by assumption} \\ &= b^{-1}a^{-1} && \text{by proposition on inverses} \end{aligned}$$

b) Prove that a and b^{-1} commute. (Hint: first show that $a = b^{-1}ab$)

c) Prove that a^2 commutes with b^2.

4. If $xay = a^{-1}$, prove that $yax = a^{-1}$.

5. Explain why in any finite group, the number of elements that are not equal to their own inverse is even. (A finite group is just a group with a finite number of elements.)

6. If G is any group, and $H = \{a \in G : a^2 = e\}$ (in other words, H is the subset of G consisting of all elements that are their own inverse), must H be a subgroup of G? If the answer is yes, give a proof. If the answer is no, give me a counterexample (that is give me a group G where H is not a subgroup).