Sierpinski Triangle (Sierpinski Gasket)

Grades: 5-8

Material: paper, ruler, protractor,crayons

The Sierpinski Triangle, created in 1916 by Waclaw Sierpinski has some very interesting properties. It is an impressive and valuable topic for mathematical exploration. It combines triangles and measurement with fractal geometry and is a popular figure to construct and analyze in middle school mathematics lessons.

Fractals are geometric shapes that are self-similar at different scales. They are formed by applying the same procedure over and over again. Sierpinski's Triangle is one of the most famous examples of a fractal although we should note that Benoit Mandelbrot first used the term fractal in 1975, almost sixty years after Sierpinski created his famous triangle.

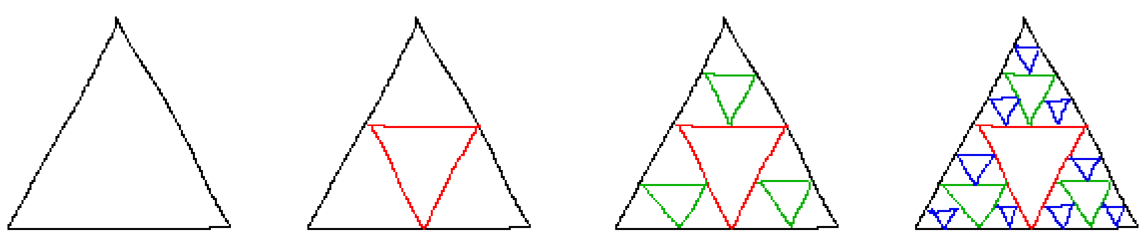
The video below gives one an idea of what a fractal is:

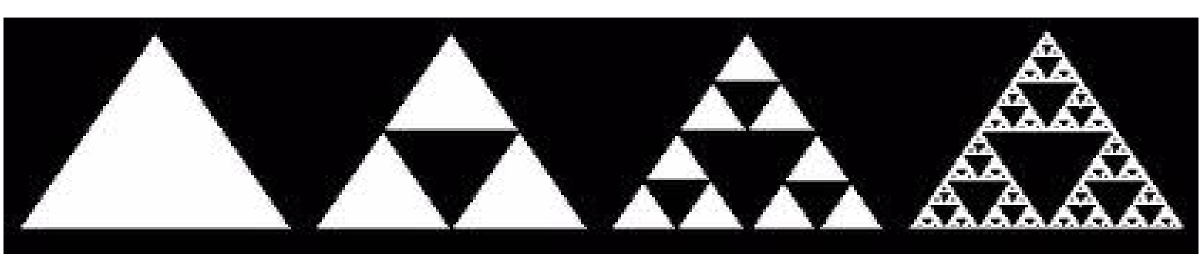
Fractal Zoom : http://www.youtube.com/watch?v=G_GBwuYuOOs

To construct a Sierpinski triangle, first draw an equilateral triangle. Determine the midpoints of each side of the triangle. Connect the midpoints with straight lines to divide the original triangle into four smaller congruent equilateral triangles. (Discuss why these smaller triangles are congruent and equilateral). Remove the middle trianlgle and repeat the same procedure for each remaining outer three triangles. Continue to repeat this entire process.

Here are two pictures that show the first four steps. In the first row you see all the triangles. When you take out the middle triangles in each step you get the second picture.

Step 0 Step 1 Step 2 Step 3





Here is a website that shows an animation of the steps of the Sierpinski Triangle:

http://www.shodor.org/interactivate/activities/SierpinskiTriangle/

In this activity, depending on the grade level of the class, you can look at properties of the triangles at each step. For example one can make a table for the number of triangles remaining at each step. One would get:

Step #	# of remaining triangles
0	1
1	3
2	9
3	27
n	?

The goal is looking at the pattern, find a relation between the step number and the number of remaining triangles. Trying some possibilities, one gets that

(Note that this formula works for all the steps in the table. That is if you plug in 0 for n, you get 1; if you plug in 1 for n you get 3; if you plug in 2 for n you get 9; and if you plug in 3 for n you get 27.

Now, having this formula at hand, ifthe question is how many triangles remain at the 5^{th} step, the answer will be (We can tell this without drawing and counting the triangles and we can do this for any step!)

For students comfortable with fractions, one can record what fraction of the area is removed at each step or what fraction of the area remains. Activity sheets are provided below.

SOLs addressed:

5.12 The student will classify

b) triangles as right, acute, obtuse, equilateral, scalene, or isosceles.

5.17 The student will describe the relationship found in a number pattern and express the relationship.

5.18 The student will
a) investigate and describe the concept of variable;
b) write an open sentence to represent a given mathematical relationship, using a variable;

6.12 The student will determine congruence of segments, angles, and polygons.

7.12 The student will represent relationships with tables, graphs, rules, and words.

7.13 The student will
b) evaluate algebraic expressions for given replacement values of the variables.

8.3 The student will
a) solve practical problems involving rational numbers, percents, ratios, and proportions;

Student Activity Sheet 1

NAME

Constructing the Sierpinski Triangle

1. Draw an equilateral triangle with sides of 32 cm. Label this triangle as step 0.

2. Determine the midpoints of each side.

3. Use these midpoints as the vertices of a new triangle, then remove the center triangle from the original triangle. This is step 1.

4. Repeat steps 2 and 3 for each remaining triangle, removing the middle triangle each time. Label the triangle accordingly.

Fractal Properties of the Sierpinski Triangle

5. Describe the procedure (recursion) to construct the Sierpinski triangle in your own words.

6. Give examples to show the self-similarity of the Sierpinski triangle.

7. Can you explain the self-similarity for the Sierpinski triangle in your own words? (Hint: remember your answer for the first question.)

Mathematical Questions

8. How many triangles remain after step 1?

9. Find the total number of triangles that remain after step 2 and after step 3.

10. Is there a pattern? Can you use it to predict the total number of triangles that will remain after step 4?

11. Can you find a formula to determine the total number of triangles that will remain after any given step?

Step #	Number of Triangles Remaining
0	
1	
2	
3	
. . .	
n	

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Student Activity Sheet 2

NAME

Area of the Sierpinski Triangle at Step n

Find the area of the Sierpinski triangle for steps 1, 2, and 3. Then discover the pattern and construct a formula for the area at any given step (step n). Use the Sierpinski triangle that you constructed for **Student Activity Sheet 1**. On each triangle, write the area that you determined for each step. For step 0, let the area of the Sierpinski triangle be 1. Answer the following questions.

1. How many triangles are there for step 1? What is the area of each of them? What is the area of the Sierpinski triangle for step 1?

2. How many triangles remain for step 2? What is the area of one of them? (Hint: use the area pattern for each of the triangles at step 1.) What is the area of the Sierpinski triangle for step 2?

3. How many triangles are left behind for step 3? What is the area of one of them? (Hint: use the area pattern for each of the triangles at step 2.) What is the area of the Sierpinski triangle for step 3?

4. What is the pattern (or relationship) between areas of the remaining triangles when moving from one step to another? State your reasoning.

5. What is the area of a remaining triangle for step n?

6. What is the formula for the area of the Sierpinski triangle for step n?

Step #	Area of the Sierpinski Triangle
0	
1	
2	
3	
. . .	
n	

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