MATH 149 CALCULUS I
SOLUTIONS TO HOMEWORK 10

2.6.18. The radius r increasing at a rate of 2in/min
Find the rate of change of volume when r = 6 in
r = 24 in

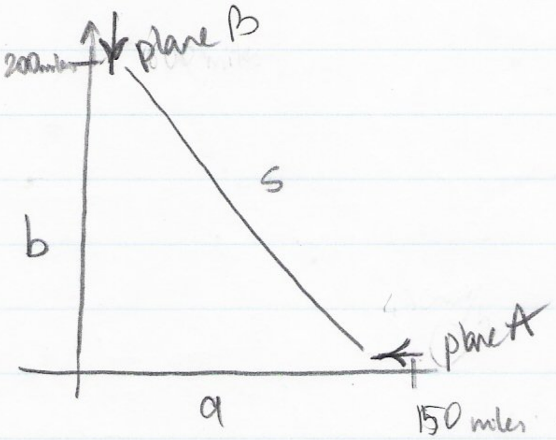
$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$$

given $\frac{dr}{dt} = 2$ in/min so $\frac{dV}{dt} = 8\pi r^2$ in^3/min

at $r = 6$, $\frac{dV}{dt} = 8\cdot\pi\cdot 6^2 = 8\pi\, 36 = 288\pi$ in^3/min

at $r = 24$, $\frac{dv}{dt} = 8\pi \cdot 24^2 = 8\cdot\pi\cdot 576$ in^3/min
$= 4608\pi$ in^3/min

2.6.31.



s = distance between two planes at time t

a = distance of plane A from the intersection

b = distance of plane B " "

Given:

$$\frac{da}{dt} = -450 \text{ miles/hour}, \quad \frac{db}{dt} = -600 \text{ miles/hour}$$

a) To be determined: $\frac{ds}{dt}$

Equation: $s^2 = a^2 + b^2$

take derivative with respect to time

$$2s\frac{ds}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$$

$$\frac{ds}{dt} = \frac{2a.(-450) + 2b(-600)}{2s}$$

So, $\frac{ds}{dt} = \frac{a}{s}(-450) + \frac{b}{s}(-600)$

at the point $a = 150$, $b = 200$,
we have $s = \sqrt{150^2 + 200^2} = 250$

So, $\frac{ds}{dt} = \frac{150}{250}(-450) + \frac{200}{250}(600)$

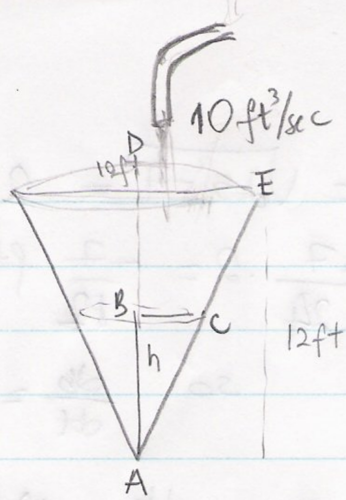
$= -270 + (-480)$

$= -750$ miles/hour.

b) 250 miles and decreasing by -750 miles/hour

$\Rightarrow$ he has 20 minutes!

2.6.24.



$$\frac{dV}{dt} = 10\ \text{ft}^3/\text{sec}$$

$$\frac{dh}{dt} = ? \text{ when } h = 8\text{ft}.$$

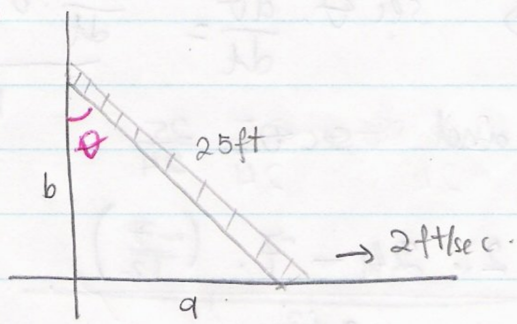
$$V = \frac{\pi r^2 h}{3}$$

$\overset{\triangle}{ABC} \sim ADE \Rightarrow \frac{h}{12} = \frac{r}{5} \Rightarrow r = \frac{5h}{12}$

So, $V = \frac{\pi \cdot \left(\frac{5h}{12}\right)^2 \cdot h}{3} \Rightarrow V = \frac{25\pi}{144 \cdot 3} h^3$

$$\underbrace{\frac{dV}{dt}}_{10} = \frac{25\pi}{432} \cdot 3\underbrace{h^2}_{8} \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{10 \cdot 432}{25\pi \cdot 3 \cdot 8^2} = \frac{9}{10}\ \text{ft}^3/\text{sec}$$

2.6.27.



$$\frac{da}{dt} = 2\ \text{ft/sec}$$

$$\frac{db}{dt} = ?$$

Pythagorean theorem: $a^2 + b^2 = 25^2$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$\frac{db}{dt} = -\frac{a}{b}\frac{da}{dt}$$

when $a = 7\,ft$, $b = \sqrt{25^2 - 7^2} = 24\,ft$

so $\frac{db}{dt} = \frac{-7}{24} \cdot 2 = \frac{-7}{12}$ ft/sec

$a = 15\,ft \Rightarrow b = 20\,ft$. so $\frac{db}{dt} = \frac{-15}{20} \cdot 2 = \frac{-3}{2}$ ft/sec

$a = 24\,ft \Rightarrow b = 7\,ft$ so $\frac{db}{dt} = \frac{-24}{7} \cdot 2 = \frac{-48}{7}$ ft/sec

b) area of the triangle $= A = a \cdot b$

$\Rightarrow \frac{dA}{dt} = \frac{da}{dt} \cdot b + a \cdot \frac{db}{dt}$

$a = 7 \Rightarrow b = 24$ and $\frac{db}{dt} = \frac{-7}{12}$ ft/sec

So, $\frac{dA}{dt} = 2 \cdot 24 + 7 \cdot \frac{-7}{12} = \frac{527}{24}$ ft^2/sec.

c) $\tan\theta = \frac{a}{b} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{\frac{da}{dt} \cdot b - a \cdot \frac{db}{dt}}{b^2}$

when $a = 7$, $b = 24$ and $\sec\theta = \frac{25}{24}$

so, $\left(\frac{25}{24}\right)^2 \frac{d\theta}{dt} = \frac{2 \cdot 24 - 7 \cdot \left(\frac{-7}{12}\right)}{24^2}$

$\frac{d\theta}{dt} = \frac{1}{12}$ radians/sec

Note: For a function to have a derivative at a point, it has to be continuous at that point.

Section 2.1 #94

$$f(x) = \begin{cases} x & \text{if } x \le 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$$

First $f(x)$ is continuous at $x=1$ because

$$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} x = 1 \\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+} x^2 = 1 \end{aligned}\right\} \text{equal and } f(1)=1 \text{ so}$$

$f(x)$ is continuous at $x=1$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{f(1+h)-1}{h}$$

$$\lim_{h\to 0^-} \frac{f(1+h)-1}{h} = \lim_{h\to 0} \frac{1+h-1}{h} = \lim_{h\to 0} \frac{h}{h} = 1$$

$$\lim_{h\to 0^+} \frac{f(1+h)-1}{h} = \lim_{h\to 0} \frac{(1+h)^2-1}{h} = \lim_{h\to 0} \frac{\cancel{1}+2h+h^2-\cancel{1}}{h} = \lim_{h\to 0} 2+h = 2$$

these are not equal so, $f'(1)$ does NOT exist

#95

$$f(x) = \begin{cases} x^2+1, & x \le 2 \\ 4x-3, & x > 2 \end{cases}$$

$\lim_{x\to 2^-} f(x) = 2^2+1 = 5$ and $\lim_{x\to 2^+} f(x) = 4\cdot 2-3 = 5$ and $f(2)=5$

so $f(x)$ is continuous at 2.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{f(2+h)-5}{h}$$

$$\lim_{h\to 0^-} \frac{f(2+h)-5}{h} = \lim_{h\to 0^-} \frac{(2+h)^2+1-5}{h} = \lim_{h\to 0^-} \frac{4+4h+h^2+1-5}{h} = \lim_{h\to 0^-} \frac{h(4+h)}{h} = 4$$

$$\lim_{h\to 0^+} \frac{f(2+h)-5}{h} = \lim_{h\to 0^+} \frac{4(2+h)-3-5}{h} = \lim_{h\to 0^+} \frac{8+4h-3-5}{h} = \lim_{h\to 0^+} \frac{4h}{h} = 4$$

equal so

$$\boxed{f'(2) = 4}$$