

Section 3-1

#17. Find critical numbers of $h(x) = \sin^2 x + \cos x$ for $0 < x < 2\pi$

$$h'(x) = 2 \sin x \cdot \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0 \Rightarrow \sin x = 0 \quad 2 \cos x - 1 = 0$$

$$x = \pi, \quad \cos x = \frac{1}{2}$$

$h'(x)$ is defined for all $x \in (0, 2\pi)$

so only critical points are $x = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ in $(0, 2\pi)$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

#28. Locate absolute extrema of $f(x) = \frac{2x}{x^2+1}$ on $[-2, 2]$

a) critical numbers.

$$f'(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} = \frac{-2(x^3 - x^2 + x - 1)}{(x^2+1)^2} = \frac{-2(x-1)^2(x+1)}{(x^2+1)^2}$$

$$f'(x) = 0 \Rightarrow x = 1, x = -1 \rightarrow \text{critical numbers.}$$

$f'(x)$ is defined for all x in $[-2, 2]$ so no critical numbers from there.

b) Evaluate at critical numbers and end points.

$$f(-1) = \frac{-2}{2} = -1 \rightarrow \text{absolute minimum} \quad f(-2) = \frac{-4}{5}$$

$$f(1) = \frac{2}{2} = 1 \rightarrow \text{absolute maximum} \quad f(2) = \frac{4}{5}$$

#40. $f(x) = \sqrt{4-x^2}$ Critical numbers: $f'(x) = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$

$f'(x)$ does not exist when $x = 2, -2$ (in the domain)

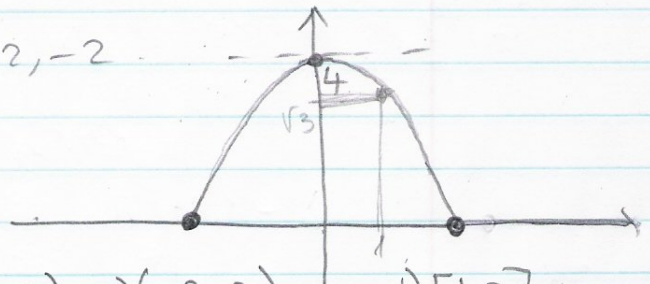
$$f'(x) = 0 \text{ when } x = 0$$

Critical numbers: $x = 0, 2, -2$

$$f(0) = \sqrt{4} = 2$$

$$f(2) = \sqrt{4-4} = 0$$

$$f(-2) = \sqrt{4-4} = 0$$



a) $[-2, 2]$ b) $[-2, 0)$ c) $(-2, 2)$ d) $[1, 2]$

absolute maximum

2 at $x=0$

does not exist

2 at $x=0$

$\sqrt{3}$ at $x=1$

absolute minimum

0 at $x=2$ and $x=-2$

0 at $x=-2$

does not exist

0 at $x=2$

#60. Cost = $C(x) = 2x + \frac{300,000}{x}$ $1 \leq x \leq 300$

critical numbers

$$C'(x) = 2 - \frac{300,000}{x^2} = 0 \Rightarrow x \approx 387 \rightarrow \text{not in } [1, 300]$$

$C(1) = 300,002$
 $C(300) = 1600$ minimum
 minimum cost of \$1600 at $x=300$

If $1 \leq x \leq 400$, then compare
 $C(1) = 300,002$, $C(387) \approx 1549.19$ minimum, $C(400) = 1550$