MATH 149 Homework 13 Solutions

Section 3.4 #16. $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$ $\Rightarrow f''(x) = 0 \Rightarrow 12x(x-2) = 0$

$x = 0,\ x = 2$

	$-\infty$	0		2	
$f''(x)$	+	o	−	o	+
$f(x)$	concave up		concave down		concave up

$x=0$ and $x=2$ are both inflection points.

#18. $f(x) = x\sqrt{x+1} \Rightarrow f'(x) = \sqrt{x+1} + x\cdot\frac{1}{2\sqrt{x+1}} = \frac{2(x+1)+x}{2\sqrt{x+1}} = \frac{3x+2}{2\sqrt{x+1}}$

Note that domain of f is $x+1 \geq 0$, $x \geq -1$

$$f''(x) = \frac{3\cdot(2\sqrt{x+1}) - (3x+2)\cdot 2\cdot\frac{1}{2\sqrt{x+1}}}{(2\sqrt{x+1})^2} = \frac{6\sqrt{x+1} - \frac{3x+2}{\sqrt{x+1}}}{4(x+1)}$$

$$= \frac{6(x+1) - 3x - 2}{4(x+1)\sqrt{x+1}} = \frac{3x+4}{4(x+1)\sqrt{x+1}}$$

$4(x+1)\sqrt{x+1}$ → positive when $x \geq -1$

$3x+4 = 0 \Rightarrow x = -\frac{4}{3} < -1$ so not in domain.

When $x \geq -1$, $3x+4$ is positive, so $f''(x)$ is positive for all x in the domain. That is, $f(x)$ is concave up in its domain.

#20. $f(x) = \frac{x+1}{\sqrt{x}} = x^{-1/2}(x+1) = x^{1/2} + x^{-1/2}$

Note that domain is $x > 0$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$, $f''(x) = -\frac{1}{4}x^{-3/2} + \frac{3}{4}x^{-5/2}$

$f''(x) = -\frac{1}{4x^{3/2}} + \frac{3}{4x^{5/2}} = \frac{-x+3}{4x^{5/2}}$

$-x+3 = 0 \Rightarrow x = 3$

$4x^{5/2} = 0 \Rightarrow x = 0.$

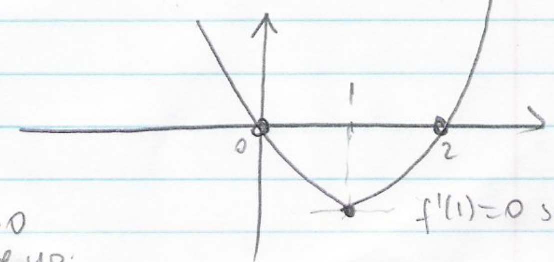
	0		3		∞
f''		+	o	−	
f		concave up		concave down	

$\Rightarrow$ $x=3$ is an inflection point

#56. $f(0) = f(2) = 0$

	$-\infty$	1	∞
f'	−	o	+
f	↘	local min	↗

$f''(x) > 0$ so concave up.



$f'(1) = 0$ so horizontal tangent at $x=1$