3.4 32, 36, 40, 44
3.5 18, 30, 34, 44

MATH 149 Homework 14 SOLUTIONS

Section 3.4 #32. $f(x) = x^3 - 9x^2 + 27x$

$f'(x) = 3x^2 - 18x + 27$

critical numbers: $f'(x) = 0 \Rightarrow 3(x^2-6x+9) = 0 \Rightarrow 3(x-3)^2 = 0 \Rightarrow x = 3$.

$f''(x) = 6x - 18$. $f''(3) = 0 \Rightarrow$ Second Derivative test inconclusive

So look at First Derivative

	$-\infty$	3	∞
$f'(x)$	+	0	+
f	$\nearrow$		$\nearrow$

no local max or min.

#36. $f(x) = \sqrt{x^2+1} \Rightarrow f'(x) = \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$ critical number: $x = 0$.

$$f''(x) = \frac{1 \cdot \sqrt{x^2+1} - x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{(\sqrt{x^2+1})^2} = \frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}} = \frac{1}{(x^2+1)^{3/2}}$$

@ critical number $\Rightarrow f''(0) = \frac{1}{1} = 1 \rightarrow$ positive so by Second Derivative Test $f(x)$ has a local minimum at $x=0$.

#40. $f(x) = 2\sin x + \cos(2x)$ on $[0, 2\pi]$

$f'(x) = 2\cos x - 2\sin(2x) = 2(\cos x - \sin(2x))$

critical numbers $f'(x) = 2(\cos x - 2\sin x \cos x) = 0 \Rightarrow \cos x(1 - 2\sin x) = 0$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$1 - 2\sin x = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

$f''(x) = 2(-\sin x - 2\cos 2x)$

$f''(\frac{\pi}{2}) = 2(-1+2) = 2 > 0 \Rightarrow$ local min.

$f''(\frac{3\pi}{2}) = 2(+1+2) = 6 > 0 \Rightarrow$ local min

$f''(\frac{\pi}{6}) = 2(-\frac{1}{2} - 2\cdot\frac{1}{2}) < 0 \Rightarrow$ local max

$f''(\frac{5\pi}{6}) = 2(-\frac{1}{2} - 2(\frac{1}{2})) < 0 \Rightarrow$ local max.

So f has local min at $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$, local max at $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$.

Section 3.5 #18

a) $\lim_{x\to\infty} \frac{3-2x}{3x^3-1} = \lim_{x\to\infty} \frac{x(\frac{3}{x}-2)}{x^3(3-\frac{1}{x^3})} = \lim_{x\to\infty} \frac{1}{x^2} = 0$

b) $\lim_{x\to\infty} \frac{3-2x}{3x-1} = \lim_{x\to\infty} \frac{x(\frac{3}{x}-2)}{x(3-\frac{1}{x})} = -\frac{2}{3}$.

c) $\lim_{x\to\infty} \frac{3-2x^2}{3x-1} = \lim_{x\to\infty} \frac{x^2(\frac{3}{x^2}-2)}{x(3-\frac{1}{x})} \to \infty$.

#30. $\lim_{x\to-\infty} \frac{-3x+1}{\sqrt{x^2+x}} = \lim_{x\to-\infty} \frac{x(-3+\frac{1}{x})}{\sqrt{x^2(1+\frac{1}{x})}} = \lim_{x\to-\infty} \frac{\overset{-1}{\cancel{x}}(-3+\frac{1}{x})}{\cancel{|x|}\sqrt{1+\frac{1}{x}}} = (-1)\cdot(-3) = 3.$

#34. $\lim_{x\to\infty} \cos(\frac{1}{x}) = \cos(0) = 1.$

#44. $\lim_{x\to-\infty} \frac{(3x+\sqrt{9x^2-x})(3x-\sqrt{9x^2-x})}{(3x-\sqrt{9x^2-x})} = \lim_{x\to-\infty} \frac{9x^2-(9x^2-x)}{3x-\sqrt{9x^2-x}}$

$= \lim_{x\to-\infty} \frac{x}{3x-|x|\sqrt{9-\frac{1}{x}}} = \lim_{x\to-\infty} \frac{\cancel{x}}{\cancel{x}(3-\frac{|x|}{x}\sqrt{9-\frac{1}{x}})} = \frac{1}{3+1\cdot\sqrt{9}}$

$= \boxed{\frac{1}{6}}$