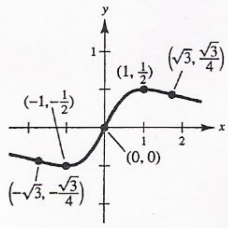
MATH 149 HOMEWORK 15 SOLUTIONS

8. $y = \dfrac{x}{x^2 + 1}$



$$y' = \frac{1 - x^2}{(x^2 + 1)^2} = \frac{(1 - x)(x + 1)}{(x^2 + 1)^2} = 0 \text{ when } x = \pm 1.$$

$$y'' = -\frac{2x(3 - x^2)}{(x^2 + 1)^3} = 0 \text{ when } x = 0, \pm\sqrt{3}.$$

Horizontal asymptote: $y = 0$

	y	y'	y''	Conclusion
$-\infty < x < -\sqrt{3}$		$-$	$-$	Decreasing, concave down
$x = -\sqrt{3}$	$-\frac{\sqrt{3}}{4}$	$-$	0	Point of inflection
$-\sqrt{3} < x < -1$		$-$	$+$	Decreasing, concave up
$x = -1$	$-\frac{1}{2}$	0	$+$	Relative minimum
$-1 < x < 0$		$+$	$+$	Increasing, concave up
$x = 0$	0	$+$	0	Point of inflection
$0 < x < 1$		$+$	$-$	Increasing, concave down
$x = 1$	$\frac{1}{2}$	0	$-$	Relative maximum
$1 < x < \sqrt{3}$		$-$	$-$	Decreasing, concave down
$x = \sqrt{3}$	$\frac{\sqrt{3}}{4}$	$-$	0	Point of inflection
$\sqrt{3} < x < \infty$		$-$	$+$	Decreasing, concave up

10. $y = \dfrac{x^2 + 1}{x^2 - 9}$

$$y' = \frac{-20x}{(x^2 - 9)^2} = 0 \quad \text{when } x = 0.$$

$$y'' = \frac{60(x^2 + 3)}{(x^2 - 9)^3} < 0 \text{ when } x = 0.$$

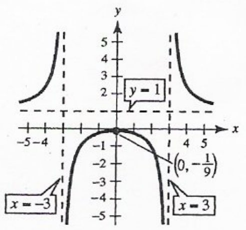
Therefore, $\left(0, -\frac{1}{9}\right)$ is a relative maximum.

Intercept: $\left(0, -\frac{1}{9}\right)$

Vertical asymptotes: $x = \pm 3$

Horizontal asymptote: $y = 1$

Symmetric about y-axis



28. $f(x) = \frac{1}{3}(x - 1)^3 + 2$

$f'(x) = (x - 1)^2 = 0$ when $x = 1$.

$f''(x) = 2(x - 1) = 0$ when $x = 1$.

	$f(x)$	$f'(x)$	$f''(x)$	Conclusion
$-\infty < x < 1$		$+$	$-$	Increasing, concave down
$x = 1$	2	0	0	Point of inflection
$1 < x < \infty$		$+$	$+$	Increasing, concave up