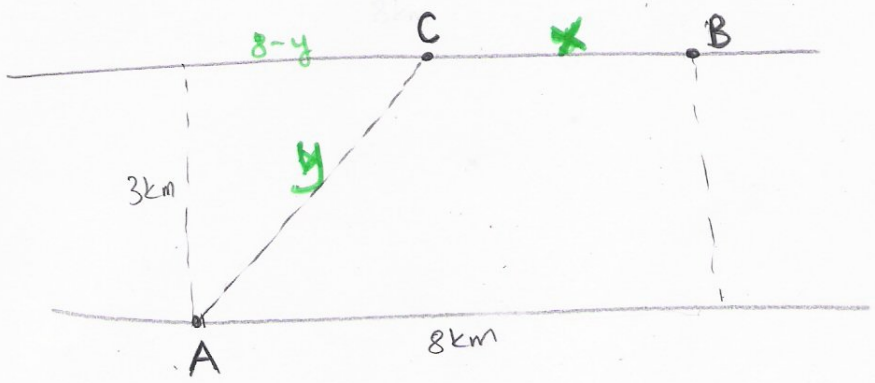
MATH 149 Homework 10 Solutions

6. A man launches his boat from point A on a bank of a straight river, 3km wide, and wants to reach point B, 8km downstream on the opposite bank, **as quickly as possible**. If he can row 6km/h and run 8km/h what should he do? (Assume the water is calm and its flow speed is negligible)



Suppose the man rows to point C then runs to point B. Let y be the distance he rows and x be the distance he runs.

The time it will take him to get to point B is: $6x+8y$.

Objective: Minimize $T = \frac{y}{6} + \frac{x}{8}$

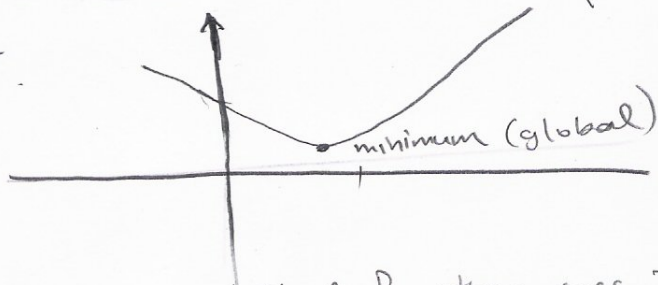
Constraint: $y^2 = (8-x)^2+3^2 \Rightarrow y = \sqrt{(8-x)^2+9}$

Then $T = \frac{\sqrt{(8-x)^2+9}}{6} + \frac{x}{8}$ and domain is $[0, 8]$ (Think about why?)

$$T' = \frac{1}{6}\cdot\frac{2(8-x)\cdot(-1)}{2\sqrt{(8-x)^2+9}} + \frac{1}{8} \Rightarrow T' = -\frac{1}{6}\cdot\frac{(8-x)}{\sqrt{(8-x)^2+9}} + \frac{1}{8}$$

$T' = 0 \Rightarrow x \cong 4.6$km. which is in the domain. (Found using DERIVE)

Sketch the graph of T.



So, if the man rows first to 4.6km left of B, then runs then it will take the least time which is 1.33 hours.