6. A man launches his boat from point A on a bank of a straight river, 3km wide, and wants to reach point B, 8km downstream on the opposite bank, as quickly as possible. If he can row 6km/h and run 8km/h what should he do? (Assume the water is calm and its flow speed is negligible)

Suppose the man rows to point C then runs to point B. Let $y$ be the distance he rows and $x$ be the distance he runs.

The time it will take him to get to point B is: $6x + 8y$

Objective: Minimize $T = \frac{y}{6} + \frac{x}{8}$

Constraint: $y^2 = (8-x)^2 + 3^2 \Rightarrow y = \sqrt{(8-x)^2 + 9}$

Then $T = \frac{\sqrt{(8-x)^2 + 9}}{6} + \frac{x}{8}$ and domain is $[0, 8]$ (Think about why?)

$T' = \frac{1}{6} \cdot \frac{2(8-x)-1}{\sqrt{(8-x)^2 + 9}} + \frac{1}{8} \Rightarrow T' = -\frac{1}{6} \cdot \frac{(8-x)}{(8-x)^2 + 9} + \frac{1}{8}$

$T' = 0 \Rightarrow x = 4.6$km which is in the domain. (Found using DERIVE)

Sketch the graph of $T$.

So, if the man rows first to 4.6km left of B, then runs then it will take the least time which is 1.33 hours.