Math 149 Homework 18 Solutions

Section 4.1 #22. $\int\left(\sqrt{x}+\frac{1}{2\sqrt{x}}\right)dx=\int\left(x^{1/2}+\frac{1}{2\sqrt{x}}\right)dx$

$=\frac{x^{3/2}}{3/2}+\sqrt{x}+C=\frac{2}{3}x^{3/2}+\sqrt{x}+C$

Verify $\left(\frac{2}{3}x^{3/2}+\sqrt{x}+C\right)'$

$=\frac{2}{3}\cdot\frac{3}{2}x^{1/2}+\frac{1}{2\sqrt{x}}+0$

$=\sqrt{x}+\frac{1}{2\sqrt{x}}$

#30. $\int(2t^2-1)^2\,dt=\int(4t^4-4t^2+1)\,dt$

$=\frac{4t^5}{5}-\frac{4t^3}{3}+t+C$

Verify: $\left(4\frac{t^5}{5}-\frac{4t^3}{3}+t+C\right)'=$
$=4t^4-4t^2+1=(2t^2-1)^2$

#28. $\int\frac{x^2+2x-3}{x^4}dx=\int\left(\frac{x^2}{x^4}+\frac{2x}{x^4}-\frac{3}{x^4}\right)dx$

$=\int\left(x^{-2}+2x^{-3}-3x^{-4}\right)dx=\frac{x^{-2+1}}{-2+1}+2\frac{x^{-3+1}}{-3+1}-3\frac{x^{-4+1}}{-4+1}+C$

$=-x^{-1}-x^{-2}+x^{-3}+C=-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C$

Verify: $\left(-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C\right)'=\left(-x^{-1}-x^{-2}+x^{-3}+C\right)'=x^{-2}+2x^{-3}-3x^{-4}+0$

$=\frac{1}{x^2}+\frac{2}{x^3}-\frac{3}{x^4}=\frac{x^2+2x-3}{x^4}$

#40. $\int\sec y\,(\tan y-\sec y)\,dy=\int(\sec y\tan y-\sec^2 y)\,dy$

$=\sec y-\tan y+C$

Verify: $(\sec y-\tan y+C)'=\sec y\tan y-\sec^2 y+0=\sec y(\tan y-\sec y)$

#58 $f'(s)=6s-8s^3$, $f(2)=3$

$f(s)=\int(6s-8s^3)\,ds$

$f(s)=\frac{6s^2}{2}-\frac{8s^4}{4}+C$

$f(s)=3s^2-2s^4+C$

$f(2)=3$

$3\cdot2^2-2\cdot2^4+C=3$

$C=23$

So, $\boxed{f(s)=3s^2-2s^4+23}$

#72.

$s(t)$ = height of the rock from the ground

$s(0) = 1800\,m$

velocity $= s'(0) = 0$ (initially no speed)

acceleration $= s''(t) = -9.8\ m/sec^2$

Then, $s'(t) = \int -9.8\,dt \Rightarrow s'(t) = -9.8t + C$

$$s'(0) = 0 \Rightarrow -9.8 \times 0 + C = 0$$

$$C = 0.$$

So, $s'(t) = -9.8t$

$$s(t) = \int s'(t)\,dt = \int -9.8t\,dt$$

$$s(t) = -9.8\frac{t^2}{2} + C$$

$$s(0) = 1800 \Rightarrow -\frac{9.8 \times 0^2}{2} + C = 1800 \Rightarrow C = 1800$$

So, $\boxed{s(t) = -9.8\frac{t^2}{2} + 1800}$

When the rock hits the ground, $s(t) = 0$

So,

$$-9.8\frac{t^2}{2} + 1800 = 0 \Rightarrow \boxed{t \cong 19.67\ \text{seconds}}$$

81. (a) $v(0) = 25 \text{ km/hr} = 25 \cdot \frac{1000}{3600} = \frac{250}{36} \text{ m/sec}$

$v(13) = 80 \text{ km/hr} = 80 \cdot \frac{1000}{3600} = \frac{800}{36} \text{ m/sec}$

$a(t) = a$ (constant acceleration)

$v(t) = at + C$

$v(0) = \frac{250}{36} \Rightarrow v(t) = at + \frac{250}{36}$

$v(13) = \frac{800}{36} = 13a + \frac{250}{36}$

$\frac{550}{36} = 13a$

$a = \frac{550}{468} = \frac{275}{234} \approx 1.175 \text{ m/sec}^2$

(b) $s(t) = a\frac{t^2}{2} + \frac{250}{36}t \quad (s(0) = 0)$

$s(13) = \frac{275}{234}\frac{(13)^2}{2} + \frac{250}{36}(13) \approx 189.58 \text{ m}$

82. $v(0) = 45 \text{ mph} = 66 \text{ ft/sec}$

30 mph = 44 ft/sec

15 mph = 22 ft/sec

$a(t) = -a$

$v(t) = -at + 66$

$s(t) = -\frac{a}{2}t^2 + 66t$ (Let $s(0) = 0$.)

$v(t) = 0$ after car moves 132 ft.

$-at + 66 = 0$ when $t = \frac{66}{a}$.

$s\left(\frac{66}{a}\right) = -\frac{a}{2}\left(\frac{66}{a}\right)^2 + 66\left(\frac{66}{a}\right)$

$= 132$ when $a = \frac{33}{2} = 16.5$.

$a(t) = -16.5$

$v(t) = -16.5t + 66$

$s(t) = -8.25t^2 + 66t$

(a) $-16.5t + 66 = 44$

$t = \frac{22}{16.5} \approx 1.333$

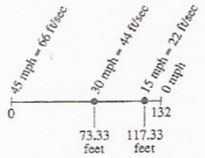
$s\left(\frac{22}{16.5}\right) \approx 73.33 \text{ ft}$

(b) $-16.5t + 66 = 22$

$t = \frac{44}{16.5} \approx 2.667$

$s\left(\frac{44}{16.5}\right) \approx 117.33 \text{ ft}$

(c)



It takes 1.333 seconds to reduce the speed from 45 mph to 30 mph, 1.333 seconds to reduce the speed from 30 mph to 15 mph, and 1.333 seconds to reduce the speed from 15 mph to 0 mph. Each time, less distance is needed to reach the next speed reduction.

83. Truck: $v(t) = 30$

$s(t) = 30t$ (Let $s(0) = 0$.)

Automobile: $a(t) = 6$

$v(t) = 6t$ (Let $v(0) = 0$.)

$s(t) = 3t^2$ (Let $s(0) = 0$.)

At the point where the automobile overtakes the truck:

$30t = 3t^2$

$0 = 3t^2 - 30t$

$0 = 3t(t - 10)$ when $t = 10$ sec.

(a) $s(10) = 3(10)^2 = 300 \text{ ft}$

(b) $v(10) = 6(10) = 60 \text{ ft/sec} \approx 41 \text{ mph}$