MATH 169 HOMEWORK 19 SOLUTIONS

Section 4.2 #10. $\left[1-\left(\frac{1}{4}\right)^2\right]+\left[1-\left(\frac{2}{4}\right)^2\right]+\cdots+\left[1-\left(\frac{4}{4}\right)^2\right]=\sum_{i=1}^{4}\left[1-\left(\frac{i}{4}\right)^2\right]$

#28. Lower sum $=\frac{1}{4}\cdot\left(f(0)+f\left(\frac{1}{4}\right)+f\left(\frac{2}{4}\right)+\cdots+f\left(\frac{7}{4}\right)\right)=$

$$=\frac{1}{4}\sum_{i=0}^{7}\left(\sqrt{\frac{i}{4}}+2\right)=5.6846$$

Upper Sum: $=\frac{1}{4}\left(f\left(\frac{1}{4}\right)+f\left(\frac{1}{2}\right)+\cdots+f\left(\frac{8}{4}\right)\right)=\frac{1}{4}\sum_{i=1}^{8}\left(\sqrt{\frac{i}{4}}+2\right)=6.0382$

#30. Lower sum $\frac{1}{5}\left(f\left(\frac{1}{5}\right)+f\left(\frac{2}{3}\right)+\cdots+f\left(\frac{5}{5}\right)\right)=\frac{1}{5}\sum_{i=1}^{5}\sqrt{1-\left(\frac{i}{5}\right)^2}=0.65926$

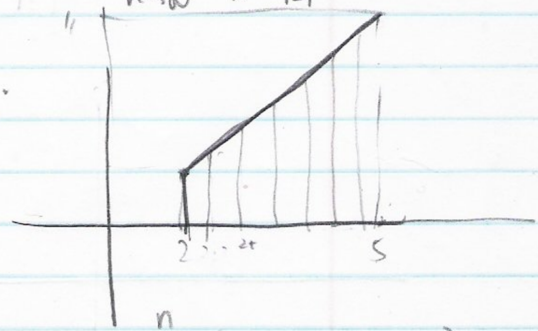
Upper sum $\frac{1}{5}\left(f(0)+f\left(\frac{1}{5}\right)+\cdots+f\left(\frac{4}{5}\right)\right)=\frac{1}{5}\sum_{i=0}^{4}\sqrt{1-\left(\frac{i}{5}\right)^2}=0.85926$.

#32. $\lim_{n\to\infty}\frac{64}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)=\frac{64\times 2}{\cancel{6}\,3}=\frac{64}{3}$ (look at the dominant terms both in numerator & denominator)

#40. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{4i}{n^2}=\lim_{n\to\infty}\frac{4}{n^2}\sum_{i=1}^{n}i$

$$=\lim_{n\to\infty}\frac{4}{n^2}\,\frac{n(n+1)}{2}=\frac{4}{2}=2.$$

#48. $y=3x-4$ on $[2,5]$



Divide into n subintervals $\to$ base $=\frac{3}{n}$

$$\lim_{n\to\infty}\underbrace{\sum_{i=1}^{n}f\left(2+i\,\frac{3}{n}\right)\cdot\frac{3}{n}}_{\text{upper sum}}=\lim_{n\to\infty}\sum_{i=1}^{n}\left(3\left(2+\frac{3i}{n}\right)-4\right)\frac{3}{n}$$

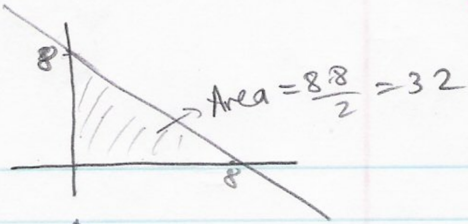
$$=\lim_{n\to\infty}\frac{3}{n}\left(\sum_{i=1}^{n}\left(6+\frac{9i}{n}-4\right)\right)=\frac{3}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\left(2+\frac{9i}{n}\right)$$

$$=\lim_{n\to\infty}\frac{3}{n}\left(\sum_{i=1}^{n}2+\frac{9}{n}\sum_{i=1}^{n}i\right)=\lim_{n\to\infty}\frac{3}{n}\left(2n+\frac{9}{n}\,\frac{n(n+1)}{2}\right)$$

$$=\lim_{n\to\infty}6+\frac{27(n^2+n)}{2n^2}=6+\frac{27}{2}=\underline{19.5}$$

(You could work with the lower sum; you would get the same result)

Section 4.3 #28. $\int_0^8 (8-x)\,dx = 32$

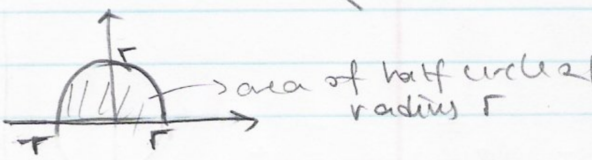


#32. $\displaystyle\int_{-r}^{r} \sqrt{r^2-x^2}\,dx = \frac{\pi r^2}{2}$

y

$y = \sqrt{r^2-x^2}$

$y^2 = r^2 - x^2 \Rightarrow y^2 + x^2 = r^2$



#40. Given $\int_2^4 x^3\,dx = 60$, $\int_2^4 x\,dx = 6$, $\int_2^4 dx = 2$

$$\int_2^4 (6+2x-x^3)\,dx = 6\int_2^4 dx + 2\int_2^4 x\,dx - \int_2^4 x^3\,dx$$

$$= 6\cdot 2 + 2\cdot 6 - 60 = 12+12-60 = -36$$

#44. Given: $\int_{-1}^{1} f(x)\,dx = 0$ $\quad \int_0^1 f(x)\,dx = 5$

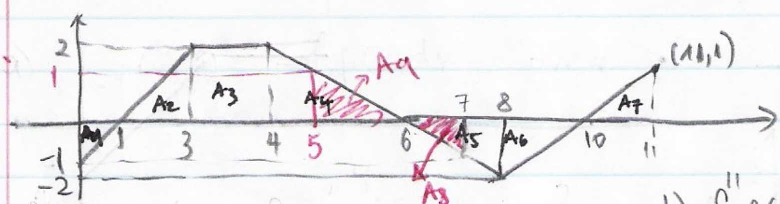
a) $\int_{-1}^{0} f(x)dx = \int_{-1}^{1} f(x)dx + \int_{1}^{0} f(x)\,dx = 0 - 5 = -5$

b) $\int_0^1 f(x)\,dx - \int_{-1}^{0} f(x)\,dx = 5 - (-5) = 10$

c) $\int_{-1}^{1} 3f(x)\,dx = 3\int_{-1}^{1} f(x)dx = 3\cdot 0 = 0$

d) $\int_0^1 3f(x)\,dx = 3\int_0^1 f(x)\,dx = 3\cdot 5 = 15$

#48.



a) $\int_0^1 -f(x)dx = -\int_0^1 f(x)dx = A_1 = \frac{1}{2}$

b) $\int_3^4 3f(x)dx = 3\int_3^4 f(x)dx = 3A_3 = 3\cdot 2 = 6$

c) $\int_0^7 f(x)dx = -A_1 + A_2 + A_3 + A_4 - A_8 = -\frac{1}{2} + 2 + 2 + 2 - \frac{1}{2} = 5$

d) $\int_5^{11} f(x)dx = A_9 - A_5 - A_6 + A_7$
$= \frac{1}{2} - 4 + \frac{1}{2} = -3$

e) $\int_0^{11} f(x)dx = -A_1 + A_2 + A_3 + A_4 - A_5 - A_6 + A_7$
$= -\frac{1}{2} + 2 + 2 + 2 - 4 + \frac{1}{2} = 2$

f) $\int_4^{10} f(x)dx = A_4 - A_5 - A_6 = 2 - 4 = -2$