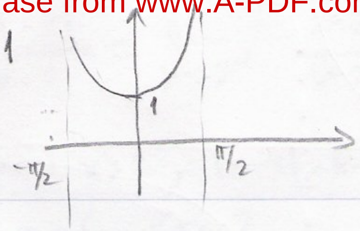
1.2 #16. $\lim_{x \to 0} \sec x = 1$

because

$\lim_{x \to 0^-} \sec x = \lim_{x \to 0^+} \sec x = 1$



#22. The points that need to be investigated are $x = -4$, $x = -2$, and $x = 0$.

At all other points the function has a limit

At $x = -4$, $\lim_{x \to -4^-} f(x) = \lim_{x \to -4^+} f(x) = 2$ so limit exists.

At $x = -2$ $\lim_{x \to -2^-} f(x) \to -\infty$ so limit does not exist.

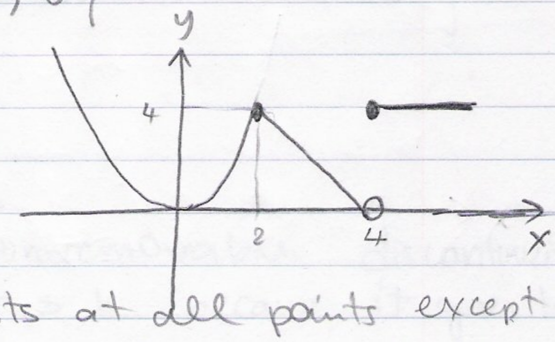
At $x = 0$ $\lim_{x \to 0^-} f(x) = 0$ which is not equal to $\lim_{x \to 0^+} f(x) = 5$

so $\lim_{x \to 0} f(x)$ does not exist.

So, $\lim_{x \to c} f(x)$ exists for all $\mathbb{R} - \{-2, 0\}$

#23.

$$f(x) = \begin{cases} x^2, & x \le 2 \\ 8 - 2x, & 2 < x < 4 \\ 4, & x \geq 4 \end{cases}$$



From the graph, limit exists at all points except at $x = 4$

because $\lim_{x \to 4^-} f(x) = 0$ whereas $\lim_{x \to 4^+} f(x) = 4$.

1.3 #14. $\lim_{x \to -3} \frac{2}{x+2} = \frac{2}{-3+2} = -2$

#30. $\lim_{x \to 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = 1$

#20. $\lim_{x \to 4} \sqrt[3]{x+4} = \sqrt[3]{4+4} = \sqrt[3]{8} = 2$

#42. $h(x) = \frac{x^2 - 3x}{x}$ $\lim_{x \to -2} h(x) = -5$, $\lim_{x \to 0} h(x) = -3$

$$\frac{x^2 - 3x}{x} = \frac{x(x-3)}{x} = x - 3 \quad \text{when } x \neq 0.$$

So, $f(x) = x - 3$ agrees with $h(x)$ at every point except at $x = 0$

1.4 #12. $\lim\limits_{\substack{x\to 2^+ \\ x>2}} \dfrac{|x-2|}{x-2} = \lim\limits_{x\to 2^+} \dfrac{x-2}{x-2} = 1$

since $x>2$, $|x-2| = x-2$.

#16. $f(x) = \begin{cases} x^2-4x+6, & x<2 \\ -x^2+4x-2, & x\geq 2 \end{cases}$

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} x^2-4x+6 = 2^2-4\cdot 2+6 = 2 \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} -x^2+4x-2 = -2^2+4\cdot 2-2 = 2 \end{aligned}\right\} \text{equal so } \lim_{x\to 2} f(x) = 2.$$

#18. $f(x) = \begin{cases} x, & x\leq 1 \\ 1-x & x>1 \end{cases}$ $\quad \lim\limits_{\substack{x\to 1^+ \\ x>1}} f(x) = \lim\limits_{x\to 1^+} 1-x = 0$

#52. For this problem you had to first do the reading assignment (Section 1.4)

$$f(x) = \tan \frac{\pi x}{2}$$

The function $\tan x$ has non-removable discontinuities at $x = k\cdot\frac{\pi}{2}$ for odd integers k because it goes to ∞ or $-\infty$ if you approach $k\cdot\frac{\pi}{2}$ from left or right.

Since $f(x) = \tan\left(\frac{\pi}{2}x\right)$, it will have non-removable discontinuities when x is an odd integer.