MATH 149 SOLUTIONS TO HOMEWORK 5

1.4 #24 $\lim_{x\to 1} \left(1 - [\![-\frac{x}{2}]\!]\right)$

plug in $x=1 \Rightarrow 1 - [\![-\frac{1}{2}]\!] = 2$

not an integer so won't jump

$\Rightarrow \lim_{x\to 1}\left(1-[\![\frac{x}{2}]\!]\right) = 2$

#42. $f(x) = \dfrac{x-1}{x^2+x-2}$

discontinuous when denominator $=0$

$x^2+x-2 = (x+2)(x-1) \Rightarrow x=-2,\ x=1$

$\lim_{x\to -2} f(x) = \lim_{x\to -2} \dfrac{x-1}{(x+2)(x-1)} \to \dfrac{1}{0} \to \infty, -\infty$

non removable

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{\cancel{(x-1)}}{(x+2)\cancel{(x-1)}} = \dfrac{1}{3}$

both left and right limits are $\frac{1}{3}$, so removable discontinuity

#46. $f(x) = \begin{cases} -2x+3, & x<1 \\ x^2, & x \geq 1 \end{cases}$

Definitely continuous except at $x=1$.

At $x=1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} -2x+3 = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = 1$

$f(1) = 1^2 = 1$

they all are equal to the same number so continuous at $x=1$.

Hence, $f(x)$ is continuous at all real numbers.

#49. $f(x) = \begin{cases} \tan\frac{\pi x}{4}, & |x|<1 \\ x, & |x|\geq 1 \end{cases} = \begin{cases} \tan\frac{\pi x}{4} & \text{if } -1<x<1 \\ x & \text{if } x<-1 \text{ or } x>1 \end{cases}$

Note that $\tan\frac{\pi x}{4}$ is continuous when $-1<x<1$ because $-\frac{\pi}{4} < \frac{\pi x}{4} < \frac{\pi}{4}$ ($\tan$ is discontinuous at $(2k+1)\frac{\pi}{2}$)

So we need to check the limits and the value of the function at $x=1$ and $x=-1$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \tan\frac{\pi x}{4} = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x = 1$

$f(1) = 1$

$f(x)$ is continuous at $x=1$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1} x = -1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} \tan\frac{\pi x}{4} = -1$

$f(-1) = -1$

$f(x)$ is continuous at $x=-1$

So, f is continuous at all

#58. $g(x) = \begin{cases} \frac{4\sin x}{x} & \text{if } x<0 \\ a-2x & \text{if } x\geq 0 \end{cases}$

The point we need to check is $x=0$. At all other points it's obviously continuous.

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{4\sin x}{x} = 4$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} a-2x = a$

To be continuous at $x=0$ we need

$4 = a = f(0) \Rightarrow a=4$.

#100 Use Intermediate Value Theorem to show that among all spheres with radii in $[1,5]$, there's one with a volume of 275.

$V(r) = \frac{4}{3}\pi r^3 \to$ continuous on $[1,5]$

$V(1) = \frac{4}{3}\pi$, $V(5) = \frac{4}{3}\pi 5^3 = \frac{500}{3}\pi$

$\frac{4}{3}\pi < 275 < \frac{500\pi}{3}$ so by the Intermediate Value Theorem, there is an r between 1 and 5 such that $V(r) = 275$.

Section 2.1 #8

Find the slope of the tangent line to $g(x) = 5-x^2$ at $(2,1)$

slope $= \lim_{h\to 0} \dfrac{g(2+h)-g(2)}{h}$

$= \lim_{h\to 0} \dfrac{5-(2+h)^2-1}{h}$

$= \lim_{h\to 0} \dfrac{5-4-4h-h^2-1}{h} =$

$= \lim_{h\to 0} \dfrac{-h(4+h)}{h} = -4$

$y-1 = -4(x-2)$

$\boxed{y = -4x+9}$