MATH 150 – SOLUTIONS TO HOMEWORK 6

Sec 2.1 #22. $f(x) = \frac{1}{x^2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{h\, x^2(x+h)^2}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}\, x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

#34. $f(x) = x^3 + 2 \Rightarrow$ slope of tangent line at x $= f'(x) = 3x^2$

The line $3x - y - 4 = 0$ has slope $= 3$.

We want $3x^2 = 3 \Rightarrow x = 1$ or $x = -1$

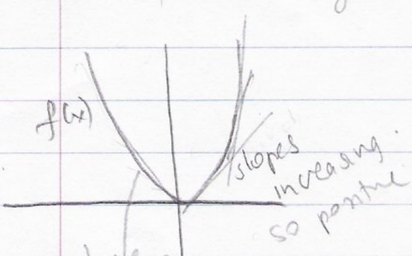
$x = 1 \Rightarrow f(x) = 1^3 + 2 = 3$

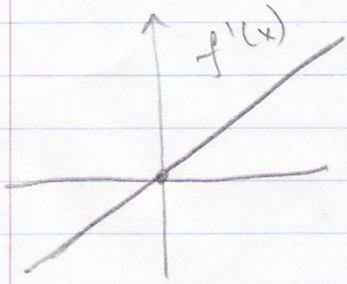
Equation of the tangent line $y - 3 = 3(x-1)$

$x = -1 \Rightarrow f(x) = (-1)^3 + 2 = 1$

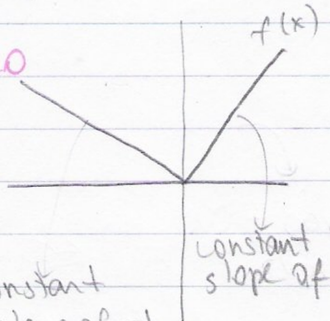
Equation of the tangent line $y - 1 = 3(x+1)$

#38





#40



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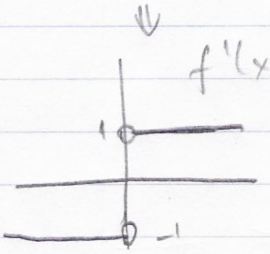
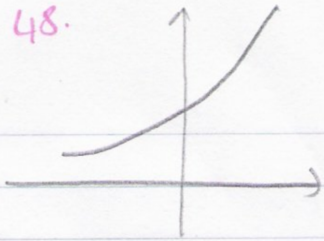


 # 48.



(Any increasing function will work)

Sec 2.2 #52

$$f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x$$

Rewrite as $f(x) = 2x^{-1/3} + 3\cos x$

Now take the derivative:

$$f'(x) = 2 \cdot \left(\frac{-1}{3}\right) x^{-4/3} - 3\sin x$$

$$= \frac{-2}{3} \cdot \frac{1}{\sqrt[3]{x^4}} - 3\sin x$$

#62. $y = \sqrt{3}\, x + 2\cos x$

$0 \le x < 2\pi$.

Where are the horizontal tangents?

Where the derivative is zero:

$y' = \sqrt{3} - 2\sin x$

$y' = 0 = \sqrt{3} - 2\sin x$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$.

#76. $f(x) = x^5 + 3x^3 + 5x$ does not have a tangent line with slope 3 because if it did we would have

$f'(x) = 5x^4 + 9x^2 + 5 = 3$

$$\Rightarrow \underbrace{5x^4 + 9x^2}_{\text{always non-negative}} = \underbrace{-2}_{\text{negative}}$$

Which can't be true.

Sec 2.3 #12. $f(t) = \frac{\cos t}{t^3}$

$$f'(t) = \frac{-\sin t \cdot t^3 - \cos t \cdot 3t^2}{(t^3)^2}$$

$$f'(t) = \frac{-t^2(t\sin t + 3\cos t)}{t^6}$$

$$f'(t) = -\frac{(t\sin t + 3\cos t)}{t^4}$$

#52. $f(x) = \sin x \cos x$

$$f'(x) = \cos x \cos x + \sin x(-\sin x)$$

$$f'(x) = \cos^2 x - \sin^2 x$$

#76. $f(x) = \frac{x-4}{x^2-7}$

For a horizontal tangent at x we should have

$$f'(x) = 0$$

$$\frac{1\cdot(x^2-7) - (x-4)\cdot 2x}{(x^2-7)^2} = 0$$

$$\frac{x^2-7-2x^2+8x}{(x^2-7)^2} = 0$$

$$\frac{-(x^2-8x+7)}{(x^2-7)^2} = 0$$

$$x^2-8x+7 = 0$$

$$x = \frac{8 \pm \sqrt{64-4\cdot1\cdot7}}{2\cdot1}$$

$$x = \frac{8 \mp \sqrt{36}}{2} \Rightarrow x = \frac{8 \mp 6}{2} \Rightarrow \boxed{x=7 \text{ or } x=1}$$