MATH149 Homework 7 Solutions

Section 2.2 #93. a) position function $= s(t) = -16t^2 + 1362$ (because $v_0 = 0$ and $s_0 = 1362$ ft)

velocity function $= s'(t) = -32t$

b) Average velocity on $[1,2] = \frac{\text{distance between } t=1 \text{ and } t=2}{\text{change in time}} = \frac{s(2)-s(1)}{2-1} = \frac{1298-1346}{1} = -48$ ft/s

c) Instantaneous velocity $= s'(t)$ $\quad t=1 \Rightarrow s'(1) = -32 \cdot 1 = -32$ ft/s

$t=1 \Rightarrow s'(2) = -32 \cdot 2 = -64$ ft/s

d) The coin reaches the ground when $s(t)=0 \Rightarrow -16t^2+1362 = 0$

$\Rightarrow t = 9.226$

e) Velocity of the coin at impact $= s'(9.226) = 32(9.226) \cong 295$ ft/sec.

#106. Inventory cost $= C = \frac{1,008,000}{Q} + 6.3Q$ $\quad Q$: order size

change in annual cost from $Q=350$ to $Q=351$ $\Rightarrow C(351) - C(350) = 5083.09 - 5085 \cong -1.91$ \$/order

instantaneous rate of change $= C'(Q) = \frac{-1,008,000}{Q^2} + 6.3$

$C'(350) \cong -1.93$ \$/order

Section 2.3 #34. $g(x) = x^2\left(\frac{2}{x} - \frac{1}{x+1}\right) = 2x - \frac{x^2}{x+1}$

$g'(x) = 2 - \frac{2x(x+1) - x^2 \cdot 1}{(x+1)^2} = 2 - \frac{2x^2+2-x^2}{(x+1)^2} = 2 - \frac{x^2+2}{(x+1)^2}$

#48. $y = \frac{\sec x}{x} \Rightarrow y' = \frac{\sec x \tan x \cdot x - \sec x \cdot 1}{x^2} = \frac{\sec x (x \tan x - 1)}{x^2}$

#87. Population of bacteria $= P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$

rate of growth $= P'(t) = 500\left(0 + \frac{4 \cdot (50+t^2) - 4t \cdot 2t}{(50+t^2)^2}\right)$

$= 500 \cdot \left(\frac{200+4t^2-8t^2}{(50+t^2)^2}\right) = 500\left(\frac{200-4t^2}{(50+t^2)^2}\right)$

rate of growth when $t=2 = P'(2) = 500\left(\frac{200 - 4 \cdot 2^2}{(50+2^2)^2}\right)$

$= 31.55$ bacteria/hour