Section 9.4 #10 \[ 0 < \frac{1}{\sqrt{n^2+1}} < \frac{1}{n^{3/2}} \]

So \[ \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \]

is convergent by p-test.

\[ p = \frac{3}{2} > 1 \]

So by Direct Comparison Test, \( \sum \frac{1}{n^{3/2}} \) is convergent.

\[ \sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5} \]

Dominant power in numerator \[ \frac{n}{n^2} \rightarrow \frac{1}{n} \]

denominator.

\[ \lim_{n \to \infty} \frac{5n-3}{n^2-2n+5} \]

\[ \lim_{n \to \infty} \frac{5n-3}{n^2-2n+5} = \lim_{n \to \infty} \frac{n}{n^2-2n+5} = \frac{1}{\infty} = 0 \]

since \( \sum \frac{1}{n} \) is divergent, \( \sum \frac{5n-3}{n^2-2n+5} \) is divergent by limit comparison test.

#28 \[ \sum_{n=1}^{\infty} \tan \frac{1}{n} \]

Compare with \( \sum_{n=1}^{\infty} \frac{1}{n} \)

\[ \lim_{n \to \infty} \tan \frac{1}{n} \]

L'Hopital's Rule

\[ \lim_{x \to 0} \frac{\tan x}{x} = 1 \]

since \( \sum \frac{1}{n} \) is divergent, by limit comparison test.

\( \sum \tan \frac{1}{n} \) is also divergent.

Section 9.5 #14 \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \]

alternating series \( a_n = \frac{1}{\ln(n+1)} \)

\[ \text{since } \ln(n+2) > \ln(n+1), \text{ we have } \frac{1}{\ln(n+2)} < \frac{1}{\ln(n+1)} \]

that is \( a_{n+1} < a_n \)

\[ \lim_{n \to \infty} \frac{1}{\ln(n+1)} = 0 \]

By alternating series test \( \sum \frac{(-1)^n}{\ln(n+1)} \) is convergent.
Solutions 9.5 #26. \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \) is alternating with \( a_n = \frac{1}{(2n+1)!} \).

Since \( \frac{1}{(2n+1)!} > \frac{1}{(2n+3)!} \),

This is true because \((2n+3)! > (2n+1)\!),

So \( a_{n+1} < a_n \).

\( \lim_{n \to \infty} \frac{1}{(2n+1)!} = 0 \).

So by Alternating Series Test \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \) is convergent.

#50 \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^n} \] check for absolute convergence,

\[ \sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n^n} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \to \text{convergent by p-test} \]

\( p = \frac{3}{2} > 1 \)

Since \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^n} \) is absolutely convergent, it is convergent itself.

#52 \[ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{n+10} \] By n-th term test, \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{n+10} \) is divergent.

#59 \[ \sum_{n=0}^{\infty} \frac{\cos(nt)}{n+1} \]

Check for absolute convergence,

\[ \sum_{n=0}^{\infty} \left| \frac{(-1)^n}{n+1} \right| = \sum_{n=0}^{\infty} \frac{1}{n+1} \to \text{divergent (compare with } \sum_{n=1}^{\infty} \frac{1}{n} \text{)} \]

So, \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \) is not absolutely convergent.

Is it conditionally convergent? Yes, we can use alternating series test: \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \) and \( a_n = \frac{1}{n+1} \to \text{decreasing because } \frac{n+1}{n+2} < \frac{1}{n+1} \)

\( \lim_{n \to \infty} \frac{1}{n+1} = 0 \).