Chapter 9 #50.

\[ S = \mathbb{Z}, \ a \sim b \text{ if } a + b \text{ is even} \]

(i) \( a + a = 2a \) is even so \( a \sim a \) (reflexive)

(ii) \( a \sim b \Rightarrow a + b = 2k \) for some \( k \in \mathbb{Z} \)

Then \( b + a = 2k \) as well \( \Rightarrow b \sim a \) (symmetric)

(iii) \( a \sim b \text{ and } b \sim c \Rightarrow a + b = 2k_1 \) for some \( k_1 \in \mathbb{Z} \)

and \( b + c = 2k_2 \) for some \( k_2 \in \mathbb{Z} \)

Then \( a + c = 2k_1 + 2k_2 - b = 2(k_1 + k_2) \)

\( \Rightarrow a \sim c \) (transitive)

So, \( R \) is an equivalence relation.

Equivalence classes:

\( S = \{ 0, \pm 2, \pm 4, \pm 6, \ldots \} \)

\( \mathbb{Z} = \{ 1, 3, 5, 7, \ldots \} \)

#52. Prove that none of \( 11, 111, 1111, \ldots \) is a square.

Note that \( 11 \ldots 11 \equiv 10^{n-1} + 10^{n-2} + \ldots + 10 + 1 \equiv 1 \pmod{4} \)

Example:

\( 1111 \equiv 1 \pmod{4} \)

(a) \( f : \mathbb{A} \rightarrow \mathbb{B}, \ g : \mathbb{B} \rightarrow \mathbb{C} \)

\( \text{Proof: Suppose } f(x) = f(y) \text{ for some } x, y \in \mathbb{A} \)

Then \( (g \circ f)(x) = (g \circ f)(y) \)

\( \text{Since } g \circ f \text{ is } 1-1, \ x = y \)

(b) \( f \circ g = 1-1 \)

\( \text{not necessarily true.} \)

Example:

\( g \) not \( 1-1 \)

\( \text{but } g \circ f \text{ is} \)

Chapter 2 #1

Two reasons why \( \mathbb{Z} = \{ 1, 1, 3, 3, 4, 4, \ldots \} \)

is not a group under +

1) not closed: \( 1 + 3 = 4 \) not odd

2) no identity

A) \( \mathbb{Z}_5 = \{ 0, 1, 2, 3, 4 \} \)

Define \( * \) on \( \mathbb{Z}_5 \) as \( a \ast b = ab \)

Show that this operation is well defined

\( \text{Proof:} \) let \( x \in a \Rightarrow 5 \mid x - a \)

\( y \in b \Rightarrow 5 \mid y - b \)

\( \text{Goal: Show that } xy \in ab \)

\( \Rightarrow x = 5k_1 + a \)

\( y = 5k_2 + b \)

\( \Rightarrow xy = (5k_1 + a)(5k_2 + b) \)

\( = 25k_1 k_2 + 5kb + 5ka + ab \)

\( 5 \mid xy - ab \Rightarrow xy \in ab \)