MATH 250 HOMEWORK 14 SOLUTIONS

13-8 #10. \( f(x,y) = x^4 + 6xy + 10y^2 - 4y + 4 \) relative extrema?

Critical points: \( f_x = 2x + 6y = 0 \), \( f_y = 6x + 20y - 4 = 0 \) (solve together)

\[ \Rightarrow x = -6, \quad y = 2 \]

\[ f_{xx} = 2, \quad f_{yy} = 20, \quad f_{xy} = 6. \quad \text{At} \ (-6,2), \ f_{xx} > 0 \text{ and } D = f_{xx}f_{yy} - f_{xy}^2 > 0 \]

So, \((-6,2,0) \text{ is a relative minimum by Second Partial Test.}\]

\#26. \( f(x,y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1 \) relative extrema and saddle?

Critical points: \( f_x = 2y - 2x^3 = 0 \), \( f_y = 2x - 2y^3 = 0 \)

\[ \Rightarrow y = x^3 \quad \Rightarrow \quad x = y^3 = (x^3)^{\frac{1}{3}} \Rightarrow x = x^\frac{1}{3} \]

\[ \Rightarrow x - x^\frac{1}{3} = 0 \Rightarrow x(x^2 - 1) = 0 \]

\[ \Rightarrow x = 0, 1, -1 \]

So, critical points are \((0,0), (1,1), (-1,-1)\)

\[ f_{xx} = -6x^2 \quad f_{yy} = -6y^2 \quad f_{xy} = 2 \quad D = f_{xx}f_{yy} - (f_{xy})^2 \]

At \((0,0)\), \( D = -4 < 0 \Rightarrow \text{ saddle point} \)

At \((1,1)\), \( D = -6(-6) - 2^2 > 0 \quad \Rightarrow \text{ \((1,1,2) \text{ is a relative max} \)} \)

At \((-1,-1)\), \( D = -6(-6) - 2^2 > 0 \quad \Rightarrow \text{ \((-1,-1,2) \text{ is a relative max} \)} \)

\#44. If \( f \) has continuous second partial derivatives on an open region containing critical point \((a,b)\)

If \( f_{xx}(a,b) \), \( f_{yy}(a,b) \) have opposite signs,

then \( D = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy})^2 < 0 \)

negative because of opposite sign

\( \Rightarrow f \) has a saddle point at \((a,b)\)