13.8 #54.

On \( y = x + 1 \), \( 0 \leq x \leq 1 \),
\[ f'(x) = \frac{dy}{dx} = (x - (x+1))^2 = (x-1)^2 \]
maximum is \( 1 \) at \( x = 0 \), minimum is \( 0 \) at \( x = 1 \)

On \( y = -\frac{1}{2}x + 4 \), \( 0 \leq x \leq 2 \),
\[ f'(x) = \frac{dy}{dx} = \frac{1}{2}x - \frac{1}{2}(x+1)^2 = \frac{5}{2}x - \frac{1}{2} \]
maximum is \( 16 \) at \( x = 2 \), minimum is \( 0 \) at \( x = 1 \)

On \( y = -2x + 1 \), \( 1 \leq x \leq 2 \),
\[ f'(x) = \frac{dy}{dx} = (x - (2x-1))^2 = (x-2)^2 \]
maximum is \( 16 \) at \( x = 2 \), minimum is \( 0 \) at \( x = 1 \)

At the critical points when \( y = 2x \), \( f'(x) = (2x-2x)^2 = 0 \)

So, \( f'(x) \) has absolute max at 16 at \( (2,0) \) and absolute min of 0 at \( (1,2) \) and along \( y = 2x \).

#56.

On the line \( y = 1 \), \( -1 \leq x \leq 1 \),
\[ f(x) = f(x) = x^2 - 2x + 1 = 1 \]

On the curve \( y = x^3 \), \( -1 \leq x \leq 1 \),
\[ f'(x) = \frac{dy}{dx} = 2x - 2x(x^2) = 2x^3 + 2x \]
\[ f'(x) = \frac{dy}{dx} = 4x^3 - 6x^2 + 2 = 2(2x^3 - 3x^2 + 1) = 0 \Rightarrow 2(x-1)^2(2x+1) = 0 \]

So, absolute max is 1 at \( (1,1) \) and on \( y = 1 \).
absolute min is \( -\frac{11}{6} \) at \( x = -\frac{1}{2} \).

#60.

Along \( y = 0 \), \( 0 \leq x \leq 4 \),
\[ f(x) = x^2 + 5 \]
max: 21 at \( x = 4 \)
min: 5 at \( x = 0 \)

Along \( x = 4 \), \( 0 \leq y \leq 2 \),
\[ f(y) = 16 - 16y + 5 \]
\[ f(0) = 5, f(2) = 1 \Rightarrow \text{max } 5 \text{ at } y = 0, \text{ min } 11 \text{ at } y = 2 \]

Along \( y = \sqrt{x} \), \( 0 \leq x \leq 4 \),
\[ f(x) = x^2 - 4 \sqrt{x} + 5 \Rightarrow f'(x) = 2x - 6x^{\frac{1}{2}} = 0 \text{ on } [0,4] \]

So, the maximum is \( f(4) = 21 \) and the minimum is \( f(4) = -11 \).