15. #18. If $\phi$ is the $n$th root of unity, then $\phi^m = \phi^n$. Let $k = m/n$. Then $(\alpha \beta)^k = \alpha^k \beta^k = (\alpha^{m/n})^k \beta^{m/n}$.

So, $\phi$ is a kth root of unity.

Note that there are many possibilities; for $k$, smallest of which is $k = \text{lcm}(m,n)$.

16. #2a) $|\bar{z} - (1-i)| \leq 3$

b) $|\text{Arg} z| < \frac{\pi}{4} \Rightarrow -\frac{\pi}{4} < \text{Arg} z < \frac{\pi}{4}$

c) $0 < |z - 2| < 3$

d) $-1 < \text{Im} z < 1$

e) $|z| > 2$

f) $(\text{Re} z)^2 > 1$

Let $z = x + iy \Rightarrow x^2 > 1$

3. b, c, e (open)
4. b, c (domain)
5. a, c (bounded)
6. a) $|z - 1 + i| > 3$
   b) $|\text{Arg} z| = \frac{\pi}{4}$
   c) Solve $z \in \mathbb{C} | |z - 2| = 3$
   d) $|\text{Im} z| = 1$
   e) $|z| = 2$
   f) $|\text{Re} z| = 1$

7. a, b, c, d, e (regions)
8. a, e (closed)
1.6 #11. \( S = \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \right\} \)

Boundary of \( S \cup \{0\} \) because for any neighborhood around \( 0 \), we can find an \( \epsilon \)-in so that \( \epsilon \) is small enough to be in that neighborhood of 0.

Also, any neighborhood of \( \frac{1}{n} \) contains \( \frac{1}{n} \) which is in \( S \) and some numbers that are not in \( S \).

2-1 #1a) \( f(z) = 3z^2 + 5z + 1 + \frac{1}{z} \)

\[ f(z) = 3(x+iy)^2 + 5(x+iy) + i + \frac{1}{x+iy} \]

\[ = 3(x^2 - y^2 + 2xy) + 5x + 5yi + i + \frac{1}{x+iy} \]

\[ = 3x^2 - 3y^2 + 5x + 1 + \frac{(6xy + 5y + 1)i}{x+iy} \]

\[ u(x+iy) = 3x^2 - 3y^2 + 5x + 1 \]

\[ v(x+iy) = 6xy + 5y + 1 \]

\[ \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} = 2 \frac{(x^2 - y^2 + 2xy) + 3}{\sqrt{(x^2 + y^2)^2}} \]

\[ = 2 \frac{x^2y^2 + 3}{\sqrt{x^2 + y^2}} + \frac{4xy}{\sqrt{x^2 + y^2}} \]

1) \( w = f(2) = \frac{i}{2} \)

a) \( |z| = 1 \Rightarrow |\frac{1}{z}| = \frac{1}{1} \Rightarrow |w| = |\frac{1}{2}| \)

b) Recall that \( \text{Arg}(\frac{1}{2}) = -\text{Arg}(2) \)

\( \text{Arg} 2 = \theta_0 \) maps to \( \text{Arg} w = -\theta_0 \)

\( |z| = 1 \Rightarrow (x-1)^2 + y^2 = 1 \Rightarrow y^2 = 1 - (x-1)^2 \)

\( w = \frac{1}{2} = \frac{x + iy}{x^2 + y^2} \Rightarrow u = \frac{x}{x^2 + y^2} \quad v = \frac{-y}{x^2 + y^2} \)

Substituting \( y^2 = 1 - (x-1)^2 \) in \( u \), we get \( u = \frac{x}{x^2 + y^2} \)

Substituting in \( v \) doesn't yield any restrictions on \( v \).

So, if \( w = u + iv \) then \( u = \frac{1}{2} \) is the image of \( |z| = 1 \)

13b) \( xy = 1 \Rightarrow y = \frac{1}{x} \)

\( f(z) = z^2 = x^2y^2 + 2xy^2 \)

Substituting \( y = \frac{1}{x} \) in \( v \), we get \( v = \frac{2x^2 - 1}{x} \)

So, image of \( w \) with \( xy = 1 \) is \( v = 2 \).