1. a) State the definition of an open set.
   b) Must the intersection of a countable collection of open sets be open? Either give me a counterexample or a proof.
   c) Must the union of a countable collection of open sets be open? Either give me a counterexample or a proof.

2. a) State the Heine-Borel theorem.
   b) Give two examples of a non-compact set: one which meets the first of the criteria for compactness in Heine-Borel, but not the second, and one which meets the second but not the first.
   c) For each of your sets from part b), give a sequence which contains no subsequence which converges to a point of the set.

3. a) Define uniform continuity.
   b) Give an example of a function \( f : \mathbb{R} \to \mathbb{R} \) which is continuous, but not uniformly continuous. Explain why it isn’t uniformly continuous.

4. a) Give an example of a function \( f : \mathbb{R} \to \mathbb{R} \) which is continuous only at \( x = 0, x = 1, \) and \( x = 2. \)
   b) Give an example of a function \( f : \mathbb{R} \to \mathbb{R} \) which is continuous only at the irrational numbers.
   c) State a theorem about \( Df \) for a monotone functions \( f. \)
   d) Would it be possible to find examples of functions in parts a) and b) which were monotone? Explain.

5. Prove \( f(x) = x^2 \) is continuous at \( x = 1. \)

5. a) Give an example of a function \( f \) which is differentiable everywhere on \( \mathbb{R} \) whose derivative is discontinuous.
   b) Is there a function \( f \) which is differentiable on \( \mathbb{R} \) whose derivative has a jump discontinuity? Either give an example, or explain why not.

6. (Bonus) Construct a function which is discontinuous only at the dyadic rationals (recall: the dyadic rationals are rational numbers which can be written in the form \( \frac{a}{b} \) where \( b \) is a power of 2).