MATH 149 – FALL 2021 **Name:**

Practice for Exam I (September 2021)

Part A In Class

1. a) Use the limit definition to find the derivative of the function $f(x) = x^2 + x$.

b) Find the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 1$.

2. a) State the intermediate value theorem.

b) Prove that the function $f(x) = x^5 + 2x^4 - 2$ has a zero in the interval $[0, 1]$.

c) Does the function $g(x) = \frac{1}{x-1}$ have a zero somewhere in the interval $[0, 2]$?

3. Find the following limits.

a) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$

b) $\lim_{x\to 0} \frac{\sin(3x)}{2x}$

c) $\lim_{x\to 2^-} \frac{x^2+x+3}{x-2}$

d) $\lim_{x\to 2} \frac{x^2+x+3}{x-2}$

e) $\lim_{x\to 5} 3$

f) $\lim_{x\to 2-} \frac{|x-2|}{x-2}$

g) $\lim_{x\to 2} \frac{|x-2|}{x-2}$

h) $\lim_{x\to \pi} \frac{\cos x}{x}$

i) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$

4. A rock is dropped from the top of a building. From the time the rock is dropped, until the time it hits the ground, the height of the rock is given by the following formula: $h(t) = 245 - 9.8t^2$. Here, h is measured in meters, and t is measured in seconds.

a) Determine a formula for the velocity v of the rock.

b) Determine the average velocity of the rock from $t = 1$ to $t = 2$.

c) Determine the velocity of the rock at $t = 1$.

d) Determine when the rock hits the ground.

e) How fast is the rock traveling at the moment it impacts the ground (before the ground starts slowing it down)?

5. Find a and b so that the function

$$f(x) = \begin{cases} x+1 & , x < 1 \\ a & , x = 1 \\ 3x^2 + bx - 1 & , x > 1 \end{cases}$$

is continuous.

6. a) If f is a function, define what it means for f to be continous at a.

b) Can f be continuous at a and still have a limit at a? Either give an example, or explain why there isn't an example.

7. Find the derivative of each of the following.
   a) $f(x) = \sin x \cos x$
   b) $g(x) = \sin(\sin x)$
   c) $h(x) = e^{-x} + x^e$
   d) $r(t) = 2^t + t^2$
   e) $s(t) = \tan \pi x$
   f) $F(x) = e^{(e^x)}$
   g) $G(x) = \frac{\sin(x^2+1)}{\sin^2 x+1}$
   h) $H(x) = \ln(\sin x)$
   i) $I(x) = \ln(\ln(\ln x))$

8. a) If $f(x) = x^{10} + 50x + 2$, find $f^{(11)}(x)$.
   b) If $f(x) = \sin 2x$, find $f^{(100)}(x)$.

9. Find the equation of the tangent line to the curve $x^3 + y^3 = 4xy + 1$ at the point $(2, 1)$.