Math 149 – Fall 2021 **Name:**

Practice for Exam II (October, 2021)

1. An extendable ladder is proped up against a wall. The base of the ladder is hinged to the ground at a distance of 3 meters from the base of the wall. Suddenly, the catch which holds the ladder extended breaks, and the ladder begins to get shorter at the rate of 2 m/s. How fast is the upper end of the ladder sliding down the wall at the moment when the ladder is 5 m long?

2. Define the following.

a) The function f has an absolute maximum on the interval $[a, b]$ at $x = a$.

b) The function f has a relative minimum at $x = a$.

c) The value $x = a$ is a critical number for f.

d) The function f is increasing on the interval (a, b). (Remember, the definition doesn't refer to the notion of derivative at all.)

f) The function f is concave down on (a, b).

3. a) State the Extreme Value Theorem. How many hypotheses are there?

b) Give an example of a continuous function defined on a non-closed interval which has no absolute maximum.

c) Give an example of a discontinuous function defined on a closed interval which has no absolute maximum.

4. a) State the Mean Value Theorem.

b) Use the MVT to prove the following statement: If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on (a, b).

c) Which theorem did we use to prove the Mean Value Theorem?

d) Which theorem did we use to prove the theorem you mentioned in part c?

e) Did we prove the theorem you mentioned in part d?

f) If you travel exactly 120 miles in exactly 2 hours, what does the MVT allow you to conclude?

5. Consider the function

$$f(x) = \frac{x+1}{x^2}.$$

a) On what intervals is f increasing/decreasing?

b) Find any critical points and classify as local maxima, local minima or neither.

c) On what intervals is f concave up/concave down?

d) Find any points of inflection.

e) Find any asymptotes.
f) Find all intercepts.
g) Use the information from parts a-f to sketch a graph of this function.
h) Repeat parts a-g for the function $f(x) = e^{-(e^x)}$.

6. The height h in feet of flood waters of a certain river are described by the formula

$$h(t) = 8te^{\left(-\frac{t}{2}\right)}$$

where t is time in days since the beginning of the flood.
a) What is the height of the flood waters at its heighest point?
b) When are flood waters dropping most rapidly? At what rate are they dropping?

7. One meter of wire is cut into two (not necessarily equal pieces). One piece is bent into the shape of a square, and the other into the shape of a circle. How much wire should be used for the square if the total area of the circle and the square is to be a minimum?

8. A rancher wishes to enclose a rectangular pasture adjacent to a river with 600 m of fencing. The side of the rectangle touching the river does not need to be fenced. Find the dimensions of the pasture which has the largest area.

9. Use Newton's method to find the zero of the function $f(x) = x^3 + x - 1$ to the nearest 0.001, using $x_1 = 1$ as your initial guess.

10. Use differentials to estimate $\sqrt{25.5}$. Compare with the actual value computed by your calculator.

11. Compute the following antiderivatives.
a) $\int (x+1)(x-2)$ dx
b) $\int \sin x + \cos x + \sec^2 x$ dx
c) $\int e^x + 2^x$ dx
d) $\int \frac{t^2+\sqrt{t}+1}{t}$ dt

e) $\int \sqrt{2x+1}$ dx
f) $\int \cos(e^x)e^x$ dx