MATH 149 – FALL 2021 **Name:**

Answers to the Practice for Exam I (September 15, 2021)

Part A In Class

1. a) Use the limit definition to find the derivative of the function $f(x) = x^2 + x$.

Don't forget to start with the definition

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2+(x+h)-(x^2+x)}{h} \end{aligned}$$

and continue along these lines until you find that $f'(x) = 2x + 1$.

b) Find the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 1$.

The slope of the tangent line $m_{\text{tan}} = f'(1) = 2(1) + 1 = 3$. Also, the line goes through the point $(1, 2)$. With this information, we can determine that the equation of the tangent line is $y = 3x - 1$

2. a) State the intermediate value theorem.

b) Prove that the function $f(x) = x^5 + 2x^4 - 2$ has a zero in the interval $[0, 1]$.

The function f is a polynomial and hence is continuous everywhere. In particular, it is continuous on the interval $[0, 1]$. Furthermore, $f(0) = -2$, and $f(1) = 1$. We note that $-2 = f(0) \leq 0 \leq f(1) = 1$. So by the intermediate value theorem, there exists an x-value c, $0 \leq c \leq 1$ such that $f(c) = 0$.

c) Does the function $g(x) = \frac{1}{x-1}$ have a zero somewhere in the interval $[0, 2]$?

Nope. The graph of g is just the curve $y = \frac{1}{x}$ shifted one unit to the right.

3. Find the following limits. **I've just included the answers here, but on an exam, you would have to show your work.**

a) $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = 4$

b) $\lim_{x\to 0} \frac{\sin(3x)}{2x} = \frac{3}{2}$

c) $\lim_{x\to 2^-} \frac{x^2+x+3}{x-2} = -\infty$

d) $\lim_{x\to 2} \frac{x^2+x+3}{x-2}$ DNE

e) $\lim_{x\to 5} 3 = 3$

f) $\lim_{x\to 2-} \frac{|x-2|}{x-2} = -1$

g) $\lim_{x\to 2} \frac{|x-2|}{x-2}$ DNE

h) $\lim_{x\to \pi} \frac{\cos x}{x} = \frac{-1}{\pi}$

i) $\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \frac{1}{2\sqrt{2}}$

4. A rock is dropped from the top of a building. From the time the rock is dropped, until the time it hits the ground, the height of the rock is given by the following formula: $h(t) = 245 - 9.8t^2$. Here, h is measured in meters, and t is measured in seconds.

a) Determine a formula for the velocity v of the rock.

$v(t) = h'(t) = -19.6t$

b) Determine the average velocity of the rock from $t = 1$ to $t = 2$.

Average velocity $= \frac{\text{how far did the rock travel}}{\text{how long it took}} = \frac{h(2)-h(1)}{2-1} = \frac{205.8-235.2}{2-1} = -29.4\text{m/s}$

c) Determine the velocity of the rock at $t = 1$.

$v(1) = -19.6(1) = -19.6\text{m/s}$

d) Determine when the rock hits the ground.

Set $h(t) = 0$ and solve for t. You get $t = 5$.

e) How fast is the rock traveling at the moment it impacts the ground (before the ground starts slowing it down)?

$v(5) = -19.6(5) = -98$ m/s.

5. Find a and b so that the function

$$f(x) = \begin{cases} x+1 & ,x<1 \\ a & ,x=1 \\ 3x^2+bx-1 & ,x>1 \end{cases}$$

is continuous.

$a = 2$, $b = 0$.

6. a) If f is a function, define what it means for f to be continous at a.

b) Can f be continuous at a and still have a limit at a? Either give an example, or explain why there isn't an example.

Of course, the answer to this question is yes. What I meant to ask is whether f can be **discontinuous** at a and still have a limit at a. And the answer to this question is yes also. Just give me any function with a removable discontinuity at a.

7. Find the derivative of each of the following.

a) $f(x) = \sin x \cos x$

$f'(x) = -\sin^2 x + \cos^2 x$

b) $g(x) = \sin(\sin x)$

$g'(x) = \cos(\sin x)\cos x$

c) $h(x) = e^{-x} + x^e$

$h'(x) = -e^{-x} + ex^{e-1}$

d) $r(t) = 2^t + t^2$

$r'(t) = (\ln 2)2^t + 2t$

e) $s(t) = \tan \pi t$

$s'(t) = \pi \sec^2 \pi t$

f) $F(x) = e^{(e^x)}$

$F'(x) = e^{(e^x)} e^x$

g) $G(x) = \frac{\sin(x^2+1)}{\sin^2 x+1}$

$G'(x) = \frac{(\cos(x^2+1))(2x)(\sin^2 x+1) - \sin(x^2+1)(2\sin x \cos x)}{(\sin^2+1)^2}$

h) $H(x) = \ln(\sin x)$

$H'(x) = \frac{\cos x}{\sin x}$

i) $I(x) = \ln(\ln(\ln x))$

$I'(x) = \frac{1}{\ln(\ln x)} \frac{1}{\ln x} \frac{1}{x}$

8. a) If $f(x) = x^{10} + 50x + 2$, find $f^{(11)}(x)$.

$f^{(11)}(x) = 0$

b) If $f(x) = \sin 2x$, find $f^{(100)}(x)$.

$f^{(100)}(x) = 2^{100} \sin 2x$

9. Find the equation of the tangent line to the curve $x^3 + y^3 = 4xy + 1$ at the point $(2, 1)$.

t.d.b.s. "x"

$3x^2 + 3y^2y' = 4xy' + 4y$

Solving, we get $y' = \frac{4y-3x^2}{3y^2-4x}$. At the point $(2, 1)$, we get $m = \frac{(4)(1)-3(2)^2}{3(1)^2-4(2)} = \frac{8}{5}$. Using this slope and point, we get that $y = \frac{8}{5}x - \frac{11}{5}$.