MATH 149 – FALL 2021 **Name:**

Solutions for Post-Midterm II Material

1. Find the antiderivative.

a) $\int \sqrt{\sin x}(\cos x)dx$

$\frac{2}{3}(\sin x)^{\frac{3}{2}}$

b) $\int e^{(\sin x+\cos x)}(\cos x - \sin x)dx$

$e^{(\sin x+\cos x)}$

c) $\int \cos(e^x)e^x \text{ dx} = \sin(e^x) + c$

2. a) Do a right-hand sum to estimate the area under the curve $y = 1 - x^2$, above the x-axis, between $x = 0$ and $x = 1$.

There was an error in this part of the question. I meant to tell you to use 4 equal subintervals. It works out to $\frac{17}{32}$.

b) Is your estimate in a an over estimate, or an under estimate? Explain how you know.

Since the rectangles sit under the curve, your extimate will be an underestimate.

c) What is the actual area under the curve $y = 1 - x^2$, above the x-axis, between x=0 and x=1?

$\int_0^1 1 - x^2 dx = [x - \frac{1}{3}x^3]_0^1 = \frac{2}{3}$

3. a) Write in sigma-notation the sum of the first 1000 natural numbers.

$$\sum_{i=1}^{1000} i$$

b) Evaluate the sum.

$\frac{(1000)(1001)}{2} = 500500$

4. a) State the (first) fundamental theorem of calculus.

b) What theorem do we use to prove the FTC? Trace back the history of the theorems we used to prove the FTC.

The FTC is proven using the MVT, which in turn is proven using Rolle's Theorem, which in turn is proven using the EVT, which we didn't prove.

c) Evaluate $\int_1^2 2^x \text{ dx}$.

$$\int_1^2 2^x dx = [\frac{1}{\ln 2}2^x]_1^2 = \frac{2}{\ln 2}$$

1. a) State the (first) fundamental theorem of calculus.

Look it up.

b) What theorem do we use to prove the FTC? Trace back the history of the theorems we used to prove the FTC.

The EVT is used in the proof of Rolle's Theorem which is used in the proof of the MVT which is used in the proof of the FTC. We never did prove the EVT.

c) Evaluate $\int_1^2 2^x$ dx.

$\int_1^2 2^x \text{ dx} = \frac{1}{\ln 2}2^2 - \frac{1}{\ln 2}2^1 = \frac{3}{\ln 2}$

5. The vertical velocity of a projectile (in m/s) is given by the formula $v(t) = 100 - 9.8t$, where t is measured in seconds since the projectile is fired.

a) If the projectile is fired from an altitude of 30 m, find a formula for the height, h, of the projectile.

$h(t) = 100t - 4.9t^2 + 30$

b) Compute the average height of the projectile over the first 5 seconds.

Average height $= \frac{\int_0^5 h(t)dt}{5-0} \approx 239$ m

6. a) State the second fundamental theorem of calculus.

Look it up.

b) Evaluate the derivative $\frac{d}{dx}(\int_1^x \cos(\cos(t))\text{dt})$.

$\cos(\cos(x))$

11. If $\int_1^3 f(x)dx = 5$ and $\int_2^3 f(x)dx = 3$, then find $\int_1^2 f(x)$.