The Pigeonhole Principle - Exercises

1. Show that in any finite gathering of people there are at least two people who know the same number of people at the gathering.

2. Inside a 1×1 square, 101 points are placed. Show that three of them form a triangle with area less than or equal to 0.01.

3. Show that among any $n+2$ integers, either there are two whose difference is a multiple of $2n$, or whose sum is divisible by $2n$.

4. Let A be any set of 20 distinct integers chosen from the arithmetic progression $1, 4, 7, \ldots, 100$. Prove that there are two distinct members of A whose sum is 104.

5. What positive integers can be expresses as the sum of two or more consecutive positive integers? (The rst three are $3 = 1 + 2$, $5 = 2 + 3$, $6 = 1 + 2 + 3$.)

6. A point in the Cartesian coordinate plane is called a lattice point if it is of the form (m, n) where m, n are integers. For example $(1, 2)$, $(-2, 5)$, and $(-3, -1)$ are all lattice points while $(-1/2, 7)$ is not. Suppose you are given five lattice points in the plane. Show that there is a lattice point on the interior of one of the line segments joining two of these points.

7. Let a, b, c, d be integers. Show that the product

$$(b-a)(c-a)(d-a)(c-b)(d-b)(d-c)$$

is divisible by 12.

8. (Putnam1971) Let there be given nine lattice points (points with integral coordinates) in three dimensional Euclidean space. Show that there is a lattice point on the interior of one of the line segments joining two of these points.)

9. Prove that there exist integers a, b, c not all zero and each of absolute value less than one million, such that

$$|a + b\sqrt{2} + c\sqrt{3}| < 10^{-11}.$$