MATH 241 – FALL 2024 **Name:**

Exam II (Thursday, October 3)

1. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$$

a) Find the eigenvalue(s) for A.

b) For each eigenvalue, determine a basis for the eigenspace.

c) Find a basis for the column space of this matrix.

d) Find a basis for the null space of this matrix.

e) Find a basis for the row space of this matrix.

f) What is the rank of A? What does the rank of a square matrix have to do with the question of whether or not the matrix is invertible?

2. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

a) Is A invertible? If so, find the inverse. If not, explain how you know.

b) Are the columns of A linearly independent? Explain how you know.

c) Are the **rows** of A spanning? Explain how you know (remember I said rows and not columns, so there's an extra step here).

3. a) Say A is a 5 by 8 matrix. If the dimension of $\mathrm{Row}(A)$ is 2, what is the dimension of $\mathrm{Col}(A)$?

b) Say A is a 5 by 8 matrix. If the dimension of Row(A) is 2, what is the dimension of Nul(A)?

c) If A is an invertible matrix, what can you tell me about the eigenvalues of A?

4. Compute the determinant of $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 1 & 2 & 3 & 4 & 6 \end{bmatrix}$

5. a) If A is an $n \times n$ matrix, how does the determinant of A tell you whether or not A is invertible?

b) If A and B are invertible $n \times n$ matrices, prove that AB is invertible. (Don't let the word "prove" scare you. And don't worry if you haven't "memorized" a proof. Just use your answer to part a. and use a property of determinants. You can do it.)