

Review For Exam III (April 23, 2003)

This exam will concentrate on section 13.1 through 14.1.

Integration of Scalar Functions

Single Integrals

$\int_a^b 1 dx$	length of interval $[a,b]$
$\int_a^b \sqrt{1 + [f'(x)]^2} dx$	length of the curve $y = f(x)$ between $x = a$ and $x = b$
$\int_a^b f(x) dx$	area under the curve $y = f(x)$, above the x-axis, between $x = a$ and $x = b$ (provided $f(x) \geq 0$ on $[a,b]$)
$m = \int_a^b \rho(x) dx$	total mass of wire with density function ρ between $x = a$ and $x = b$
$\bar{x} = \frac{\int_a^b x\rho(x) dx}{m}$	coordinate of center of mass of wire with density function ρ between $x = a$ and $x = b$

Double Integrals

$\iint_R 1 dA$	area of region R
$\iint_R \sqrt{1 + [f_x(x, y)]^2 + [f_y(x, y)]^2} dA$	surface area of the surface given by $z = f(x, y)$ over the region R
$\iint_R f(x, y) dA$	volume under the surface $z = f(x, y)$, above the xy-plane, over the region R (provided $f(x, y) \geq 0$ on R)
$m = \iint_R \rho(x, y) dA$	total mass of lamina with density function ρ over the region R
$\bar{x} = \frac{\iint_R x\rho(x, y) dA}{m}$	x coordinate of center of mass of lamina with density function ρ over the region R

Triple Integrals

$\iiint_Q 1 dV$	volume of solid region Q
$\iiint_Q \sqrt{1 + [f_x(x, y, z)]^2 + [f_y(x, y, z)]^2 + [f_z(x, y, z)]^2} dV$	requires 4 dimensions to interpret geometrically
$\iiint_Q f(x, y, z) dV$	requires 4 dimensions to interpret geometrically
$m = \iiint_Q \rho(x, y, z) dV$	total mass of solid with density function ρ over the region Q
$\bar{x} = \frac{\iiint_Q x\rho(x, y, z) dV}{m}$	x coordinate of center of mass of solid with density function ρ over the region Q

Some other things from Chapter 13 you should practice:

Doing double integrals in polar coordinates.

Setting up a triple integral.

Converting a triple integral into either cylindrical or spherical coordinates.

Computing a Jacobian.

Stuff from section 14.1:

Be able to sketch a simple vector field in \mathbf{R}^2 .

State the definition of a conservative vector field.

Use the definition to decide whether or not a vector field is conservative.

Be able to compute divergence and curl.

Understand when you may use Theorems 14.1 and 14.2 to determine whether or not a vector field is conservative, and be able to use them when appropriate.

Problem: Let $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$.

a) Does $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$?

b) Is \mathbf{F} conservative?

c) Explain what is going on here.