Math 250 – Spring 2011 **Name:**

Review for Exam II

1. Consider the function $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$.
   a) Find the arc length of the curve defined by $\mathbf{r}$ on the interval $(0, 2)$.
   b) Find the curvature of the curve defined by $\mathbf{r}$ at $t = 1$.

2. a) Find the limit, or explain why it doesn't exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

b) Give an example of a function $f : \mathbf{R}^2 \to \mathbf{R}$ which is discontinuous at $(x, y) = (0, 0)$, but the limit of f exists at $(0, 0)$.

3. Let $f(x, y) = e^{\frac{x}{y}}$
   a) Compute f_{xy} and f_{yx}.
   b) State conditions on a function which will guarantee that the mixed partials will equal.
   c) Give an example of a function where the mixed partial derivatives do not equal.

4. Let $z = \sqrt{x^2 - e^x}$.
   a) Compute the differential dz.
   b) Use part a to help estimate the value of $\sqrt{25.1 - e^{.1}}$.

5. The radius of a circular cylinder is increasing at a rate of 2 meters per second, while the height is decreasing at a rate of 3 meters per second. Find the rate of change of volume when the height is 10 m, and the radius is 20 m.

6. Find and classify all critical points of the function $f(x, y) = 2x^3 + 2y^3 - 3x^2 - 3y^2$.

7. The pressure P in kilopascals at sea level at a point (x,y) (where distance is measured in kilometers and the origin is at the center of a hurricane) is given by the function

$$P(x, y) = 101.325 - 15e^{-(x^2+y^2)}.$$

a) Find the rate of change of pressure at the point (1, 1) in the direction of the vector $\langle 2, -1 \rangle$.

b) Sketch the isobar (curve of constant pressure) at pressure 100 kilopascals. What shape are the other isobars?

c) What does your answer to b imply about the direction of the gradient vectors? Do these vectors point towards the origin, or away from it? Explain.