MATH 328 – SPRING 2020

Final Exam

Show all work.

You may use the table for y_p in the method of undetermined coefficients, and the table of Laplace transforms.

You may work on your own paper.

Part A: You must complete this part

1. Write the first five terms of the Taylor series centered at $x = 0$ for the function $f(x) = e^{-x}$.

2. Use Laplace transforms to solve the differential equation $y'' + y = 0$, $y(0) = 3$, $y'(0) = 2$.

Part B: You may complete this part if you choose

3. Solve the following differential equations and initial value problems.

a) $y'' - 4y' + 13y = 0$

b) $y'' + y = \sin t$

c) $y'' - 3y' = 0$, $y(0) = 1$, $y'(0) = 2$

d) $y^{(7)} - 11y^{(6)} + 53y^{(5)} - 143y^{(4)} + 231y''' - 221y'' + 115y' - 25y = 0$ (Please feel free to use electronic means to factor your characteristic polynomial.)

Part C: You may complete this part if you choose

4. Solve the following differential equations and initial value problems.

a) $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

b) $(3xy + y^2) + (x^2 + xy)y' = 0$

c) $y' + \frac{1}{t}y = \cos t$

5. Consider the differential equation $y' = y^3 - 2y^2 + y$

a) Find and discuss the stability of the equilibrium points.

b) Plot a slope field and discuss and sketch 3 different-looking solution curves.

6. A 1000 liter container initially contains a saline solution of conentration 10 grams of salt per liter. The container is flushed with pure water at a rate of 3 liters/min while solution is drawn off at the same rate. The solution in the tank is assumed to be well-mixed.

a) Write and solve a differential equation for the amout of salt in the take at a given time.

b) When will the concentration of salt in the tank drop to 1 gram per liter?