

Problems

1. In year zero, we begin with a single pair of newborn rabbits, one male and one female. A pair of rabbits gives birth to one pair of rabbits, one male and one female, every year, beginning at one year of age. These rabbits are immortal.
 - a) Find the number of **pairs of** rabbits for the first eight years.
 - b) Conjecture a formula for the number of pairs of rabbits after n years.
 - c) Prove the formula you conjectured in part b) as best you can.
2. Repeat problem 1 except a pair of rabbits gives birth to one pair of baby rabbits every year beginning at age two years.
3. Repeat problem 1 including both the following two differences:
 - i. In year zero, we introduce one pair of baby rabbits, and in year one, we introduce two more pairs of baby rabbits.
 - ii. A pair of rabbits gives birth to six pairs of baby rabbits every year beginning at age two years.
4. Repeat problem 3 except in year zero, we introduce three pairs of baby rabbits, and in year one, we introduce one more pair of baby rabbits.
5. What can you say about the growth rates of the populations in questions 1 to 3?
6. For each of the populations in problems 1 through 4, and for each of the years 1 through 8, compute the ratio of the number of pairs of rabbits to the number of pairs of rabbits of the previous year.
7. Of the following pairs, which grows faster in the long run as n gets larger?
 - a) $10n^2$ or n^3
 - b) n or $1000\sqrt{n}$
 - c) n^2 or 2^n
 - d) n^{10} or 2^n
 - e) $\sqrt{n}$ or $\ln(n)$