Math 443 – Assignment 1 – October 31

1. $\mathbf{N} \times \mathbf{N} = \{(a, b) : a, b \in \mathbf{N}\}$. In other words, the set of all ordered pairs of natural numbers.

Prove that $| \mathbf{N} \times \mathbf{N} | = | \mathbf{N} |$.

2. Show that all finite sequences of zeros and ones are countable.

3. Are all sequences of zeros and ones countable. Either prove that they are, or explain why they aren't.

4. Show that the extreme value theorem isn't true over the rational numbers. That is, find a continuous function on a closed interval of rational numbers with no absolute maximum and/or no absolute minimum.

5. Define a sequence of real numbers as follows: Let $x_1 = 1$, and then for every $n \in \mathbf{N}$, define $x_{n+1} = \frac{x_n}{2} + 1$.

a) Use mathematical induction to prove that $x_n < x_{n+1}$ for all $n \in \mathbf{N}$.

b) Use induction to prove that $x_n < 2$ for all $n \in \mathbf{N}$.

6. Prove that $\sqrt{3}$ is irrational. Where does a similar proof fail when trying to show that $\sqrt{4}$ is irrational?