

Math 4443 – Assignment 1 – Due: Thursday, October 28

1. Define a sequence of real numbers as follows: Let $x_1 = 1$, and then for every $n \in \mathbf{N}$, define $x_{n+1} = \frac{x_n}{2} + 1$.

a) Use mathematical induction to prove that $x_n < x_{n+1}$ for all $n \in \mathbf{N}$.

b) Use induction to prove that $x_n < 2$ for all $n \in \mathbf{N}$.

2. Prove that $\sqrt{3}$ is irrational. Where does a similar proof fail when trying to show that $\sqrt{4}$ is irrational?

3. Prove that the extreme value theorem doesn't work over $\mathbf{Q}$. That is, find a continuous function, $f : \mathbf{Q} \rightarrow \mathbf{Q}$ defined on a closed and bounded interval $I = [a, b]$ of **rational** numbers such that f has no maximum on I . Just to clarify, your job is to give me f and I .

Hint: Use the same thought process we used when we showed that the intermediate value theorem doesn't work over $\mathbf{Q}$. Just find a function that would reach its maximum at an irrational number. It's important that $f : \mathbf{Q} \rightarrow \mathbf{Q}$. So, while it's okay to use a polynomial, make sure that if you do, your coefficients are rational.