Math 4443 – Assignment 3 – Due: November 24

1. Either give an example or explain why it's impossible.

a) Sequences (x_n) and (y_n) that both diverge, but $(x_n + y_n)$ converges.

b) Sequences (x_n) and (y_n) such that (x_n) and $(x_n + y_n)$ converge, but (y_n) diverges.

c) Sequences (x_n) and (y_n) that both diverge, but $(x_n y_n)$ converges.

d) Sequences (x_n) and (y_n) such that (x_n) and $(x_n y_n)$ converge, but (y_n) diverges.

2. Consider the following two definitions:

A sequence (a_n) is *eventually* in a set $A \subseteq \mathbf{R}$ if there exists an $N \in \mathbf{N}$ such that $a_n \in A$ for all $n \geq N$.

A sequence (a_n) is *frequently* in a set $A \subseteq \mathbf{R}$ if for every $N \in \mathbf{N}$, there exists an $n \geq N$ such that $a_n \in A$.

a) Give a sequence (x_n), and a set A such that the sequence is frequently in A without eventually being in A, or explain why that's impossible.

a) Give a sequence (x_n), and a set A such that the sequence is eventually in A without frequently being in A, or explain why that's impossible.

3. a) Form a Cantor-like set, except instead of removing open middle thirds, remove open middle quarters. For instance, to go from the unit interval to the next set in your nested chain, you would remove $(\frac{3}{8}, \frac{5}{8})$. How much room does this set take up on the number line?

b) Do the same thing, but instead of removing middle thirds, remove the middle half. For instance, to go from the unit interval to the next set in your nested chain, you would remove $(\frac{1}{4}, \frac{3}{4})$. How much room does this set take up on the number line?