MATH 494 – FALL 2020 **Name:**
**Final Exam** (December 2020)

1. Consider the partition $14 = 5 + 4 + 2 + 2 + 1$.

a) [2 points] Give the Ferrer's Diagram for this partition.

b) [2 points] Find the conjugate partition.

2. a) [3 points] Show that the number of partitions of $n$ in distinct and odd parts is equal to the number of self conjugate partitions of $n$.

b) [4 points] Show that the number of partitions of $n$ in distinct and odd parts is equal to the number of self conjugate partitions of $n$.

3. a) [2 points] What is a constructible number?

b) [5 points] Prove that the constructible numbers form a field.

4. [5 points] Show that $\sqrt{5} - \sqrt[3]{2}$ is an algebraic number.

5. a) [3 points] What is the Weierstrass M-test?

b) [5 points] Use the Weierstrass M-test to prove that $1 + x + x^2 + \ldots$ converges to $\frac{1}{1-x}$ uniformly on $(-a, a)$ for any $0 < a < 1$.

6. a) [4 points] In what two ways does Niven's proof that $\pi$ is irrational resemble Euclid's proof that $\sqrt{2}$ is irrational?

b) [3 points] In the proof that $\pi$ is irrational, what definition of $\pi$ did we use?

7. a) [3 points] Demonstrate that the natural numbers $\mathbf{N} = \{1, 2, 3, \ldots\}$ and the integers $\mathbf{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ have the same cardinality.

b) [3 points] Give an example of an infinite set which doesn't have the same cardinality as $\mathbf{N}$, and name (or describe) the proof that tells you that these two sets have different cardinality.