Math 119 Homework 10 Solutions

Section 2.2 #14. $y = t^2 + 2t - 3 \Rightarrow y' = 2t + 2$

#26. $y = \dfrac{2}{3x^2} \underset{\text{rewrite}}{=} \dfrac{2}{3}x^{-2} \underset{\text{differentiate}}{\Rightarrow} y' = \dfrac{-4}{3}x^{-3} \overset{\text{simplify}}{=} \dfrac{-4}{3x^3}$

#42. $f(x) = x + \dfrac{1}{x^2} \underset{\text{rewrite}}{=} x + x^{-2} \underset{\text{differentiate}}{\Rightarrow} f'(x) = 1 - 2x^{-3} \overset{\text{simplify}}{=} 1 - \dfrac{2}{x^3}$

#48. $f(x) = \sqrt[3]{x} + \sqrt[5]{x} \underset{\text{rewrite}}{=} x^{1/3} + x^{1/5} \underset{\text{differentiate}}{\Rightarrow} f'(x) = \dfrac{1}{3}x^{-2/3} + \dfrac{1}{5}x^{-4/5} = \dfrac{1}{3x^{2/3}} + \dfrac{1}{5x^{4/5}}$

#50. $f(t) = t^{2/3} - t^{1/3} + 4 \underset{\text{differentiate}}{\Rightarrow} f'(t) = \dfrac{2}{3}t^{-1/3} - \dfrac{1}{3}t^{-2/3} + 0$

$= \dfrac{2}{3t^{1/3}} - \dfrac{1}{3t^{2/3}}$

#57. $y = x^4 - 8x^2 + 2$

horizontal tangent has slope zero.

So need to solve $y' = 0 \Rightarrow 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0 \Rightarrow x = 0, 2, -2$

#76. $f(x) = x^5 + 3x^3 + 5x$

$f'(x) = 5x^4 + 9x^2 + 5 = 3 \Rightarrow \underbrace{5x^4 + 9x^2}_{\text{always non negative b/c of } x^4 \text{ and } x^2} = \underbrace{-2}_{\text{negative}}$

So, $f'(x)$ cannot be 3.

Section 2.3 #2. $f(x) = (6x+5)(x^3-2) \Rightarrow f'(x) = 6(x^3-2) + (6x+5)\cdot 3x^2$

$= 6x^3 - 12 + 18x^3 + 15x^2$

#8. $g(t) = \dfrac{t^2+2}{2t-7} \Rightarrow g'(t) = \dfrac{2t(2t-7) - (t^2+2)\cdot 2}{(2t-7)^2} = \dfrac{4t^2 - 14t - 2t^2 - 4}{(2t-7)^2}$

$= \dfrac{2t^2 - 14t - 4}{(2t-7)^2}$

#16. $f(x) = \dfrac{x+1}{x-1} \Rightarrow f'(x) = \dfrac{1\cdot(x-1) - (x+1)\,1}{(x-1)^2} = \dfrac{x-1-x-1}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

$f'(2) = \dfrac{-2}{(2-1)^2} = \boxed{-2}$

Sec. 2-2 #30 $f(x) = \sqrt[3]{x}\,(\sqrt{x}+3) = x^{1/3}(x^{1/2}+3)$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}(x^{1/2}+3) + x^{1/3}\cdot\frac{1}{2}x^{-1/2}$$

(Note: $x^{-\frac{2}{3}}\cdot x^{\frac{1}{2}} = x^{-\frac{2}{3}+\frac{1}{2}} = x^{-\frac{1}{6}}$
$x^{\frac{1}{3}}\cdot x^{-\frac{1}{2}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{-\frac{1}{6}}$)

$$= \frac{1}{3}x^{-\frac{1}{6}} + x^{-\frac{2}{3}} + \frac{1}{2}x^{-\frac{1}{6}} = \boxed{\frac{1}{6x^{1/6}} + \frac{1}{x^{2/3}}}$$

#72 $f(x) = \dfrac{4x}{x^2+6} \Rightarrow f'(x) = \dfrac{4(x^2+6) - 4x\cdot 2x}{(x^2+6)^2}$

$$\Rightarrow f'(x) = \frac{4x^2+24-8x^2}{(x^2+6)^2} = \frac{24-4x^2}{(x^2+6)^2}$$

@ $\left(x=2, y=\frac{4}{5}\right)$ slope of tangent line is $f'(2)$

$$f'(2) = \frac{24-4\cdot 2^2}{(2^2+6)^2} = \frac{24-16}{100} = \frac{8}{100} = \frac{2}{25}$$

equation of the tangent line:

$$y - \frac{4}{5} = \frac{2}{25}(x-2)$$

$$\boxed{y = \frac{2}{25}x + \frac{16}{25}}$$