MATH 149 Homework #13 Solutions

Sec 2.4 #10. $f(t) = (9t+2)^{2/3} \Rightarrow f'(t) = \frac{2}{3}(9t+2)^{-1/3} \cdot 9 = \frac{6}{\sqrt[3]{9t+2}}$

#28. $y = \frac{x}{\sqrt{x^4+4}} \Rightarrow y' = \frac{1 \cdot \sqrt{x^4+4} - x \cdot \frac{1}{2\sqrt{x^4+4}} \cdot 4x^3}{(\sqrt{x^4+4})^2}$

$$y' = \frac{2(x^4+4) - 4x^4}{2\sqrt{x^4+4}\,(x^4+4)} = \frac{8-2x^4}{2(x^4+4)^{3/2}} = \frac{4-x^4}{(x^4+4)^{3/2}}$$

#30. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2 \Rightarrow h'(t) = 2\left(\frac{t^2}{t^3+2}\right)' \cdot \frac{2t(t^3+2) - t^2 \cdot 3t^2}{(t^3+2)^2}$

$$= \frac{2t^2(2t^4+4t-3t^4)}{(t^3+2)^3} = \frac{2t^2(4t-t^4)}{(t^3+2)^3}$$

#42. $y = \sin(\pi x) \Rightarrow y' = \cos(\pi x) \cdot \pi = \pi\cos(\pi x)$

#58. $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x} \Rightarrow$ rewrite $y = \sin x^{1/3} + (\sin x)^{1/3}$

differentiate $\Rightarrow y' = \cos x^{1/3} \cdot \frac{1}{3}x^{-2/3} + \frac{1}{3}(\sin x)^{-2/3} \cdot \cos x$

$$= \frac{\cos\sqrt[3]{x}}{3\sqrt[3]{x^2}} + \frac{\cos x}{3\sqrt[3]{(\sin x)^2}}$$

#74. $y = 2\tan^3 x$; tangent line @ $\left(\frac{\pi}{4}, 2\right)$

slope formula: $y' = 2 \cdot 3\tan^2 x \cdot \sec^2 x = 6\tan^2 x \sec^2 x$

slope @ $\left(\frac{\pi}{4}, 2\right)$ is $m = 6 \cdot \tan^2\frac{\pi}{4} \cdot \sec^2\frac{\pi}{4} = 6 \cdot 1^2 \cdot (\sqrt{2})^2 = 12$

equation of the tangent line @ $\left(\frac{\pi}{4}, 2\right)$

$$y - 2 = 12\left(x - \frac{\pi}{4}\right)$$

$$\boxed{y = 12x - 3\pi + 2}$$