MATH 149 Homework 14 Solutions

Sec 2.5 #10. $2\sin x\cos y = 1$ Find $\frac{dy}{dx}$.

$$2\left(\cos x\cos y + \sin x(-\sin y)\frac{dy}{dx}\right) = 0 \Rightarrow \frac{dy}{dx} = \frac{\cos x\cos y}{\sin x\sin y}$$

#32. $x^3 + y^3 - 6xy = 0$. Find $\frac{dy}{dx}$ @ $\left(\frac{4}{3}, \frac{8}{3}\right)$

$$3x^2 + 3y^2\frac{dy}{dx} - 6\left(y + x\cdot\frac{dy}{dx}\right) = 0 \Rightarrow$$

$$\frac{dy}{dx} = \frac{-3x^2 + 6y}{3y^2 - 6x} \Rightarrow \frac{dy}{dx}\left(\frac{4}{3}, \frac{8}{3}\right) = \frac{-3\left(\frac{4}{3}\right)^2 + 6\cdot\frac{8}{3}}{3\left(\frac{8}{3}\right)^2 - 6\cdot\frac{4}{3}} = \boxed{\frac{4}{5}}$$

#38. $x^{2/3} + y^{2/3} = 5$, Find $\frac{dy}{dx}$ @ $(8,1)$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

$$\Rightarrow \frac{dy}{dx}(8,1) = -\frac{\sqrt[3]{1}}{\sqrt[3]{8}} = -\frac{1}{2}. \Rightarrow \text{tangent line: } y - 1 = -\frac{1}{2}(x-8)$$

$$\boxed{y = -\frac{1}{2}x + 5}$$

(b) #52. $y^2 = \frac{x-1}{x^2+1}$ Tangent line at $\left(2, \frac{\sqrt{5}}{5}\right)$

$$2y\frac{dy}{dx} = \frac{1\cdot(x^2+1) - (x-1)2x}{(x^2+1)^2} \Rightarrow \frac{dy}{dx} = \frac{-x^2 + 2x + 1}{(x^2+1)^2\cdot 2y}$$

$$\frac{dy}{dx}\left(2, \frac{\sqrt{5}}{5}\right) = \frac{-2^2 + 2\cdot 2 + 1}{(2^2+1)^2\cdot 2\cdot\left(\frac{\sqrt{5}}{5}\right)} = \frac{1}{10\sqrt{5}} = \frac{\sqrt{5}}{50} = \text{slope}.$$

tangent line

$$y - \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{50}(x-2)$$

$$\boxed{y = \frac{\sqrt{5}}{50}x + 2 + \frac{\sqrt{5}}{5}}$$