Section 3.4 # 16. $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 \Rightarrow f''(x) = 12x^2 - 24x = 12x(x-2)$

$f'' = 0 \Rightarrow x = 0,\ x = 2$

	0		2	
f''	+	−		+
f	concave up	concave down		concave up

At $x=0$, $x=2$ inflection points

#24. $f(x) = \sin x + \cos x$, $[0, 2\pi]$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

$f''(x) = 0 \Rightarrow \sin x = -\cos x$

$\tan x = -1$

$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

	0	$3\pi/4$	$7\pi/4$	2π
$f''(x)$	−	+	−	
f	concave down	concave up	concave down	

At $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$, inflection points

 # 34. $g(x) = -\frac{1}{8}(x+2)^2(x-4)^2$

$g'(x) = -\frac{1}{8}\left[2(x+2)(x-4)^2 + (x+2)^2 \cdot 2(x-4)\right]$

$= -\frac{2}{8}(x+2)(x-4)\left(x-4+x+2\right) = -\frac{2}{8}(x^2-2x-8)(2x-2)$

$= -\frac{1}{2}(x^2-2x-8)(x-1)$

$g'(x) = 0 \Rightarrow x = 1,\ x = 4,\ x = -2 \to$ critical numbers

Second Derivative Test

$g''(x) = -\frac{1}{2}\left[(2x-2)(x-1) + (x^2-2x-8)\cdot 1\right]$

$= -\frac{1}{2}\left[2x^2 - 3x + 2 + x^2 - 2x - 8\right] = -\frac{1}{2}\left[3x^2 - 5x - 6\right]$

plug in critical numbers in $g''(x)$ and apply the second derivative test

$g''(1) = -\frac{1}{2} \cdot -8 = 4 > 0 \Rightarrow$ local min at $x = 1$

$g''(4) = -\frac{1}{2}\left(3\cdot 4^2 - 5\cdot 4 - 6\right) = -11 < 0 \Rightarrow$ local max at $x = 4$

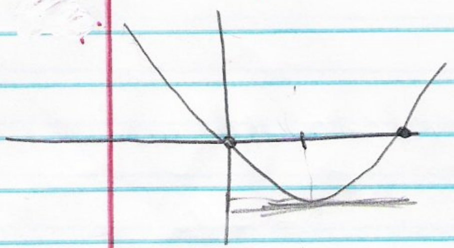
$g''(-2) = -\frac{1}{2}\left[3\cdot 4 - 5(-2) - 6\right] = -8 < 0 \Rightarrow$ local max at $x = -2$

Section 3.4 #56. $f(0) = f(2) = 0$

$f'(x) < 0$ if $x < 1$ → decreasing, $f'(1) = 0$ ⇒ horizontal tangent

$f'(x) > 0$ if $x > 1$ → increasing

$f''(x) > 0$ ⇒ concave up.



#68. Cost $= C = 2x + \dfrac{300000}{x}$ x: units

To find the minimum, find the critical numbers and make a sign chart for C': (or you can use the second derivative test)

$C' = 2 - \dfrac{300{,}000}{x^2} \Rightarrow C' = 0 \Rightarrow x = \mp\sqrt{150{,}000}$

C' undefined ⇒ $x = 0$

	0		$\sqrt{150{,}000}$		∞
C'	‖	+	0	−	
C		↗ increasing		↘ decreasing	

⇒ making a maximum at $x = \sqrt{150{,}000} \approx 387$ units