Sec 3.7 #8. Find $x, y \in \mathbb{R}^+$ such that $x^2+y=27$ and xy maximum

$x^2+y=27 \Rightarrow y=27-x^2$ $f(x)=x\cdot y = x(27-x^2)=$

$f(x)=27x-x^3$

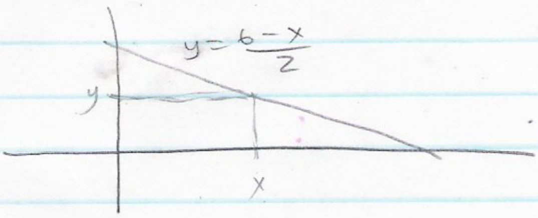
$f'(x)=27-3x^2=0 \Rightarrow x=\pm 3 \Rightarrow x=3>0$

	0		3	
f'		+	0	−
f		↗		↘

At $x=3$, $f(x)$ is maximum. $y=27-3^2 \Rightarrow \boxed{y=18}$

Maximum product is $3\times 18=54$

#24.



Area of a rectangle

$f(x)=x\cdot\frac{(6-x)}{2}=\frac{6x-x^2}{2}$, $x\geq 0$

$f'(x)=3-x$.

$f'(x)=0 \Rightarrow x=3$

x	0		3	
f'		+	0	−
f		↗		↘

max. at $x=3$, $y=\frac{6-3}{2}=\frac{3}{2}$

Max Area $=3\cdot\frac{3}{2}=\boxed{\frac{9}{2}}$

#40.

$V=3000\,ft^3=\frac{4}{3}\pi r^3+\pi r^2 h$

Surface of cylinder $=2\pi r h\ ft^2$

Surface of two hemispheres $=4\pi r^2\ ft^2$

cost
k dollars/ft^2
2k dollars/ft^2

Cost: $2k(4\pi r^2)+k\cdot(2\pi r h)$ minimize this function

Using $V=3000$, $h=\frac{3000-\frac{4}{3}\pi r^3}{\pi r^2}=\frac{3000}{\pi r^2}-\frac{4}{3}r$

The $\text{Cost}=f(r)=k\left(8\pi r^2+2\pi r\left(\frac{3000}{\pi r^2}-\frac{4}{3}r\right)\right)$

$f(r)=k\left[\frac{16}{3}\pi r^2+\frac{6000}{r}\right]$

$f'(r)=k\left[\frac{32}{3}\pi r-\frac{6000}{r^2}\right]=0 \Rightarrow \boxed{r=\sqrt[3]{\frac{1125}{2\pi}}\approx 5.636 ft}$

	0		5.636		∞
f'		−		+	
f		↘		↗	

min at $r=5.636$, then $h=\boxed{22.545 ft}$.

Sec 3.7 #60 a) Profit $= P = -\frac{1}{10}s^3 + 6s^2 + 400$

$$P' = -\frac{3}{10}s^2 + 12s = 0 \Rightarrow s\left(-\frac{3}{10}s + 12\right) = 0$$

$s = 0, \quad s = 40$

	0		40	
P'	0	+	0	−
P		↗		↘

profit = maximum at $\boxed{s = 40}$.

b) To see when the rate of growth of the profit function begins to decline we need to look at P''.

$$P'' = \frac{-6}{10}s + 12 = 0 \Rightarrow s = 20$$

	0		20	
P''		+	0	−
P'		↗		↘

rate of growth begins to decline when $s = 20$.

Sec 3.8 #10

Newton's Method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = 1 - 2x^3$

$f'(x) = -6x^2$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	1	−1	−6	0.8333
2	0.833	−0.1573	−4.1663	0.7955
3	0.7955	−0.0068	−3.7969	0.7937
4	0.7937	0.0000	−3.7798	0.7937

same

#22

$$y = 4x^3 - 12x^2 + 12x - 3 = f(x)$$

$$y' = 12x^2 - 24x + 12 = f'(x)$$

$$x_1 = \frac{3}{2}$$

$$\Rightarrow x_2 = \frac{3}{2} - \frac{f\left(\frac{3}{2}\right)}{f'\left(\frac{3}{2}\right)} = 1.$$

$f'(x_2) = f'(1) = 0 \Rightarrow$ can't continue the iteration.