1.4

#16. $f(x) = \begin{cases} x^2-4x+6, & x<2 \\ -x^2+4x-2, & x \geq 2 \end{cases}$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2-4x+6 = 2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} -x^2+4x-2 = 2$

equal so limit exists.

#28. $f(x) = \begin{cases} x & x<1 \\ 2 & x=1 \\ 2x-1 & x>1 \end{cases}$

The only point that needs to be checked is $x=1$. At all other points f is continuous.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1} x = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 2x-1 = 1$

limit exists: $\lim_{x\to 1} f(x) = 1$

but $f(1) = 2$.

Since $f(1) \neq \lim_{x\to 1} f(x)$, $f(x)$ is not continuous at $x=1$.

#40. $f(x) = \dfrac{x-3}{x^2-9}$: $f(x)$ is continuous everywhere except $x=3$ and $x=-3$.

$\lim_{x\to 3} \dfrac{x-3}{x^2-9} = \lim_{x\to 3} \dfrac{x-3}{(x-3)(x+3)} = \dfrac{1}{6}$ but $f(3)$ does not exist. (removable discontinuity)

$\lim_{x\to -3} \dfrac{x-3}{x^2-9} = \dfrac{-6}{0} \to \infty$ involved (non removable discontinuity)

#46. $f(x) = \begin{cases} -2x+3, & x<1 \\ x^2, & x \geq 1 \end{cases}$

Only point that needs to be checked is $x=1$.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} -2x+3 = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = 1$

$f(1) = 1^2 = 1$

all equal $\Rightarrow$ $f(x)$ is continuous at $x=1$

Hence, $f(x)$ is continuous at all real numbers.