Math 149 Homework 9 Solutions

Section 2.1 #14. $f(x) = 3x+2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)+2-(3x+2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x}+3h+\cancel{2}-\cancel{3x}-\cancel{2}}{h} = \lim_{h\to 0} \frac{3h}{h} = 3.$$

#18. $f(x) = 1-x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1-(x+h)^2-(1-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{1-(x^2+2xh+h^2)-1+x^2}{h} = \lim_{h\to 0} \frac{\cancel{1}-\cancel{x^2}-2xh-h^2-\cancel{1}+\cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{-h(2x+h)}{h} = -2x$$

#95. $f(x) = \begin{cases} x^2+1, & x \le 2 \\ 4x-3, & x > 2 \end{cases}$ Notice $f(2) = 5$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}.$$

Need to look at the limit from the left & from the right.

$$\lim_{h\to 0^-} \frac{f(2+h)-5}{h} = \lim_{h\to 0^-} \frac{(2+h)^2+1-5}{h} = \lim_{h\to 0^-} \frac{4+4h+h^2+\cancel{1}-\cancel{5}}{h}$$

$$= \lim_{h\to 0^-} \frac{h(4+h)}{h} = 4$$

$$\lim_{h\to 0^+} \frac{f(2+h)-5}{h} = \lim_{h\to 0^+} \frac{4(2+h)-3-5}{h} = \lim_{h\to 0^+} \frac{8+4h-3-5}{h}$$

$$= \lim_{h\to 0^+} \frac{4h}{h} = 4.$$

Because left and right limits are equal $f'(2)$ exists.