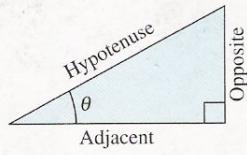


# TRIGONOMETRY

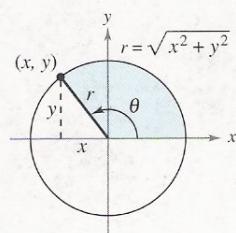
## Definition of the Six Trigonometric Functions

Right triangle definitions, where  $0 < \theta < \pi/2$ .

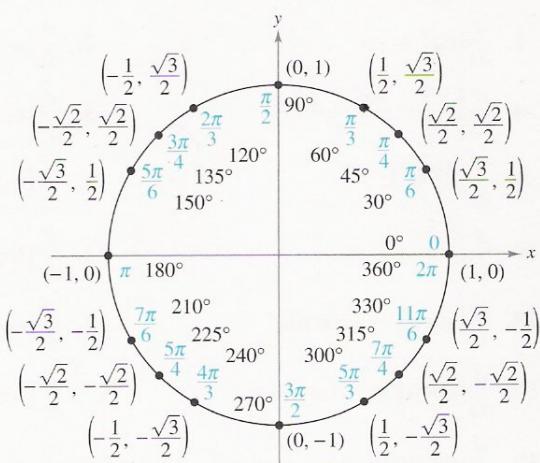


$$\begin{array}{ll} \sin \theta = \frac{\text{opp}}{\text{hyp}} & \csc \theta = \frac{\text{hyp}}{\text{opp}} \\ \cos \theta = \frac{\text{adj}}{\text{hyp}} & \sec \theta = \frac{\text{hyp}}{\text{adj}} \\ \tan \theta = \frac{\text{opp}}{\text{adj}} & \cot \theta = \frac{\text{adj}}{\text{opp}} \end{array}$$

Circular function definitions, where  $\theta$  is any angle.



$$\begin{array}{ll} \sin \theta = \frac{y}{r} & \csc \theta = \frac{r}{y} \\ \cos \theta = \frac{x}{r} & \sec \theta = \frac{r}{x} \\ \tan \theta = \frac{y}{x} & \cot \theta = \frac{x}{y} \end{array}$$



## Reciprocal Identities

$$\begin{array}{lll} \sin x = \frac{1}{\csc x} & \sec x = \frac{1}{\cos x} & \tan x = \frac{1}{\cot x} \\ \csc x = \frac{1}{\sin x} & \cos x = \frac{1}{\sec x} & \cot x = \frac{1}{\tan x} \end{array}$$

## Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

## Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x \quad 1 + \cot^2 x = \csc^2 x \end{aligned}$$

## Cofunction Identities

$$\begin{array}{ll} \sin\left(\frac{\pi}{2} - x\right) = \cos x & \cos\left(\frac{\pi}{2} - x\right) = \sin x \\ \csc\left(\frac{\pi}{2} - x\right) = \sec x & \tan\left(\frac{\pi}{2} - x\right) = \cot x \\ \sec\left(\frac{\pi}{2} - x\right) = \csc x & \cot\left(\frac{\pi}{2} - x\right) = \tan x \end{array}$$

## Reduction Formulas

$$\begin{array}{ll} \sin(-x) = -\sin x & \cos(-x) = \cos x \\ \csc(-x) = -\csc x & \tan(-x) = -\tan x \\ \sec(-x) = \sec x & \cot(-x) = -\cot x \end{array}$$

## Sum and Difference Formulas

$$\begin{array}{ll} \sin(u \pm v) = \sin u \cos v \pm \cos u \sin v & \\ \cos(u \pm v) = \cos u \cos v \mp \sin u \sin v & \\ \tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} & \end{array}$$

## Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

## Power-Reducing Formulas

$$\begin{array}{l} \sin^2 u = \frac{1 - \cos 2u}{2} \\ \cos^2 u = \frac{1 + \cos 2u}{2} \\ \tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u} \end{array}$$

## Sum-to-Product Formulas

$$\begin{array}{l} \sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{array}$$

## Product-to-Sum Formulas

$$\begin{array}{l} \sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{array}$$