5.1 #34. $\frac{3}{2}\left[\ln(x^2+1)-\ln(x+1)-\ln(x-1)\right]$

$= \frac{3}{2}\ln\frac{x^2+1}{(x+1)(x-1)} = \ln\left(\frac{x^2+1}{x^2-1}\right)^{\frac{3}{2}}$

#42. $y = \ln x^{3/2}$

Find the tangent line @ (1,0)

formula for the slope $= y' = \frac{1}{x^{3/2}} \cdot \frac{3}{2}x^{1/2}$

$y' = \frac{3}{2x}$

Then slope @ $x=1$ is $m = \frac{3}{2\cdot 1} = \frac{3}{2}$

Equation:

$y - 0 = \frac{3}{2}(x-1) \Rightarrow \boxed{y = \frac{3}{2}x - \frac{3}{2}}$

#58. $y = \ln\sqrt[3]{\frac{x-1}{x+1}}$ Find y'.

$y = \frac{1}{3}\left(\ln(x-1) - \ln(x+1)\right)$

$y' = \frac{1}{3}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$

78. $\ln(\underbrace{xy}_{\text{Product Rule}}) + 5x = 30$ Find $\frac{dy}{dx}$

Take derivatives: $\frac{1}{xy}\cdot\left(y + x\cdot\frac{dy}{dx}\right) + 5 = 0$

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = -5$

$\boxed{\frac{dy}{dx} = y\cdot\left(-5 - \frac{1}{x}\right)}$

#98. $y = \frac{(x+1)(x+2)}{(x-1)(x-2)}$

$\ln y = \ln\left(\frac{(x+1)(x+2)}{(x-1)(x-2)}\right)$

$\ln y = \ln(x+1) + \ln(x+2) - \ln(x-1) - \ln(x-2)$

Take derivatives: $\frac{y'}{y} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}$

$y' = y\cdot\left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}\right)$

$= \frac{(x+1)(x+2)}{(x-1)(x-2)}\left(\frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x-1} - \frac{1}{x-2}\right)$

38 $\lim_{x\to 6^-}\ln(6-x) \to -\infty$

Since $x \to 6$, $6-x$ goes to zero but because $x \to 6^-$ x is a little less than 6 so $6-x$ is positive (but very close to zero)

so $6 - x \to 0^+$

$\ln(x)$ goes to $-\infty$ as $x \to 0^+$

#~~40~~ $x + y - 1 = \ln(x^2+y^2)$

Find the tangent line at (1,0)

Implicit differentiation

$1 + y' = \frac{1}{x^2+y^2}(2x + 2yy')$

Plug in $x=1$, $y=0$ and solve for y'. That will be the slope

$1 + y' = \frac{1}{1+0}(2+0)$

$y' = 2 - 1 = 1 = m$

Equation $y - 0 = 1(x-1)$

$\boxed{y = x - 1}$

5.2 #20 $\int \frac{1}{x\ln(x^3)}dx = I$

$= \int \frac{1}{x\,3\ln x}dx = \frac{1}{3}\int\frac{1}{x\ln x}dx$

Substitute $u = \ln x$

$du = \frac{1}{x}dx$

$= \frac{1}{3}\int\frac{1}{u}du = \frac{1}{3}\ln|u| + C$

~~#28~~ $= \frac{1}{3}\ln|\ln(x)| + C$

~~#34~~

5.2. #28 $\int \frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\,dx$ (bonus)

Substitute $u = \sqrt[3]{x} - 1$

$du = \frac{1}{3}x^{-2/3}\,dx$

$3 \cdot x^{2/3}\,du = dx$

$$\Rightarrow \int \frac{x^{1/3}}{u}\, 3 \cdot x^{2/3}\,du$$

$$\Rightarrow 3\int \frac{x}{u}\,du$$

$u = \sqrt[3]{x} - 1$

$\Rightarrow u+1 = \sqrt[3]{x}$

$(u+1)^3 = x$

$$= 3 \cdot \int \frac{(u+1)^3}{u}\,du$$

$$= 3\int \frac{u^3+3u^2+3u+1}{u}\,du = 3\int \left(u^2+3u+3+\frac{1}{u}\right)du$$

$$= 3\left(\frac{u^3}{3}+\frac{3u^2}{2}+3u+\ln|u|\right)+C$$

$$= (\sqrt[3]{x}-1)^3 + \frac{9}{2}(\sqrt[3]{x}-1)^2 + 9(\sqrt[3]{x}-1) + \ln|\sqrt[3]{x}-1| + C$$

5.2 #36 $\int (\sec t + \tan t)\,dt$

$$= \int \frac{1}{\cos t} + \frac{\sin t}{\cos t}\,dt$$

$$= \int \sec t\,dt + \int \frac{\sin t}{\cos t}\,dt$$

$$= \ln|\sec t + \tan t| - \ln|\cos t| + C$$