MATH 150 HW 3 SOLUTIONS

Sect. 5.5 # 44) $h(x) = \log_3 \frac{x\sqrt{x-1}}{2}$

Find $h'(x)$.

$$h(x) = \log_3 x + \frac{1}{2}\log_3(x-1) - \log 2$$

$$h'(x) = \frac{1}{x\ln 3} + \frac{1}{2(x-1)\ln 3} - 0$$

52) $y = \log_{10}(2x)$, tangent line @ $(5,1)$

$$y' = \frac{1}{2x(\ln 10)} =$$

slope at $x = 5$: $m = \frac{1}{10(\ln 10)}$

$$y - 1 = \frac{1}{10\ln 10}(x-5)$$

$$\boxed{y = \frac{1}{10\ln 10}x - \frac{1}{2\ln 10} + 1}$$

68)
$$\int_{-2}^{2} 4^{x/2}\,dx = \int_{-2}^{2} (2^2)^{x/2}\,dx$$

$$= \int_{-2}^{2} 2^x\,dx = \frac{2^x}{\ln 2}\Bigg|_{-2}^{2}$$

$$= \frac{2^2}{\ln 2} - \frac{2^{-2}}{\ln 2} = \frac{4}{\ln 2} - \frac{1}{4\ln 2}$$

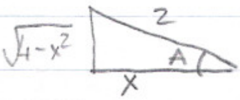
$$\boxed{= \frac{15}{4\ln 2}}$$

5.6 # 30) $f(x) = \tan\big(\underbrace{\arccos\frac{x}{2}}_{A}\big)$

$g(x) = \frac{\sqrt{4-x^2}}{x}$

→ $\arccos\frac{x}{2} = A \Rightarrow \cos A = \frac{x}{2}$

(calculated using Pythagorean Theorem) ←



Then $\tan A = \frac{\sqrt{4-x^2}}{x} = g(x)$.

See attached for DERIVE

58) $y = 25\arcsin\frac{x}{5} - x\sqrt{25-x^2}$

Find y'.

$$y' = 25\cdot\frac{1}{\sqrt{1-\left(\frac{x}{5}\right)^2}}\cdot\frac{1}{5} - \Big[x\cdot\frac{-2x}{2\sqrt{25-x^2}} + \sqrt{25-x}$$

$$y' = \frac{5}{\sqrt{1-\frac{x^2}{25}}} + \frac{x^2}{\sqrt{25-x^2}} - \sqrt{25-x^2}$$

$$y' = \frac{5}{\sqrt{\frac{25-x^2}{25}}} + \frac{x^2-(25-x^2)}{\sqrt{25-x^2}}$$

$$y' = \frac{25}{\sqrt{25-x^2}} - \frac{25+2x^2}{\sqrt{25-x^2}} = \boxed{\frac{2x^2}{\sqrt{25-x^2}}}$$

75) $x^2 + x\arctan y = y - 1$

Find the tangent line at $\left(\frac{-\pi}{4}, 1\right)$

Differentiate both sides:

$$2x + \arctan y + x\cdot\frac{1}{1+y^2}\cdot\frac{dy}{dt} = \frac{dy}{dt}$$

$$\left(\frac{x}{1+y^2} - 1\right)\frac{dy}{dt} = -2x - \arctan y$$

Substitute $x = -\frac{\pi}{4}$, $y = 1$ and solve for $\frac{dy}{dt}$ to find the slope:

$$\Rightarrow \quad m = \frac{-2\pi}{\pi+8}$$

$$y - 1 = -\frac{2\pi}{\pi+8}\left(x + \frac{\pi}{4}\right)$$

$$\boxed{y = -\frac{2\pi x}{\pi+8} - \frac{2\pi^2}{4(\pi+8)} + 1}$$

Section 5.7.

#12) $\int \frac{1}{x\sqrt{x^4-4}}\,dx$

$u=x^2 \Rightarrow du=2x\,dx$

write as

$$=\int \frac{x}{x^2\sqrt{x^4-4}}\,dx$$

$$=\int \frac{1}{u\sqrt{u^2-2^2}}\cdot\frac{du}{2}$$

$$=\frac{1}{2}\int \frac{1}{u\sqrt{u^2-2^2}}\,du$$

$$=\frac{1}{2}\,\frac{1}{2}\,\text{arcsec}\,\frac{|u|}{2}+C$$

$$\boxed{=\frac{1}{4}\,\text{arcsec}\,\frac{x^2}{2}+C}$$

#18) $\int \frac{4x+3}{\sqrt{1-x^2}}\,dx=\int \frac{4x}{\sqrt{1-x^2}}\,dx+\int \frac{3}{\sqrt{1-x^2}}\,dx$

$u=1-x^2$

$du=-2x\,dx$

$$=\int \frac{-2}{\sqrt{u}}\,du+3\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$=-2\,\frac{u^{1/2}}{1/2}+3\arcsin x+c$$

$$\boxed{=-4\sqrt{1-x^2}+3\arcsin x+C}$$

#30) $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

Substitute
$u=\sin x$
$du=\cos x\,dx$

bounds
$x=0\Rightarrow u=0$
$x=\frac{\pi}{2}\Rightarrow u=1$

$$\int_0^1 \frac{du}{1+u^2}=\arctan u\Big|_0^1$$

$$=\arctan 1-\arctan 0$$

$$=\frac{\pi}{4}-0=\boxed{\frac{\pi}{4}}$$

#32) $\int_{-2}^{2} \frac{dx}{x^2+4x+13}$

complete the square in the denominator

$(x^2+4x+4)+9$

$(x+2)^2+9$

So

$$\int_{-2}^{2} \frac{dx}{(x+2)^2+9}$$

substitute
$u=x+2\Rightarrow du=dx$

bounds
$x=-2\Rightarrow u=0$
$u=2\Rightarrow u=4$

So,

$$\int_0^4 \frac{du}{u^2+3^2}=\frac{1}{3}\arctan\frac{u}{3}\Big|_0^4$$

$$=\frac{1}{3}\arctan\frac{4}{3}-\underbrace{\frac{1}{3}\arctan\frac{0}{3}}_{0}$$

$$\boxed{=\frac{1}{3}\arctan\frac{4}{3}}$$