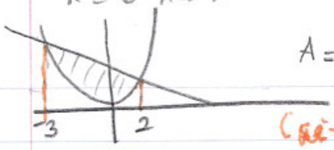
MATH 150 HW4 SOLUTIONS

7.1 #14. Area between $y=x^2$ and $y=6-x$

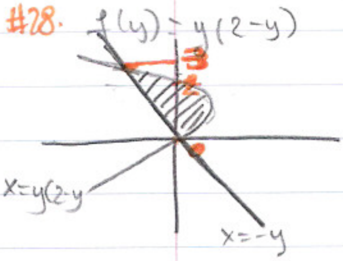
$x^2=6-x \Rightarrow x^2+x-6=0 \Rightarrow x=2,\ x=-3$



$A=\int_{-3}^{2}\left[(6-x)-x^2\right]dx=\boxed{\frac{125}{6}}$

(with respect to y → see below on the right)

#28. $f(y)=y(2-y)$ $g(y)=-y \Rightarrow f(y)=g(y)$



$2y-y^2=-y \Rightarrow$

$0=y^2-3y$

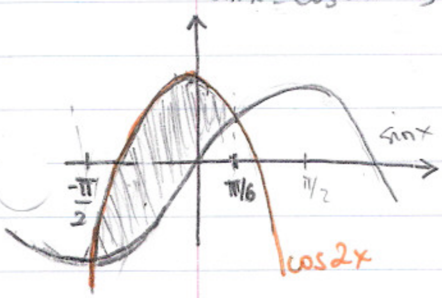
$y=0,\ y=3$

$A=\int_0^3\left[(2y-y^2)-(-y)\right]dy$

$=\int_0^3\left[3y-y^2\right]dy=\boxed{\frac{9}{2}}$

#44. $f(x)=\sin x$ $g(x)=\cos 2x$ $\frac{-\pi}{2}\le x\le\frac{\pi}{6}$

$\sin x=\cos 2x \Rightarrow x=-\frac{\pi}{2}$ and $x=\frac{\pi}{6}$



$A=\int_{-\pi/2}^{\pi/6}(\cos 2x-\sin x)\,dx$

$=\frac{1}{2}\sin 2x+\cos x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{6}}=\boxed{\frac{3\sqrt{3}}{4}\approx 1.3}$

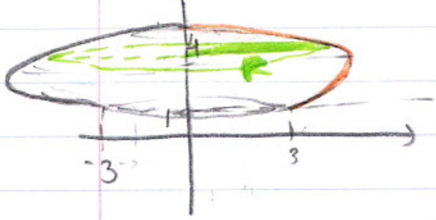
#50. $f(x)=2\sin x+\cos(2x)$, $y=0$, $0<x\le\pi$

a) graph (see attached)

b) $\int_0^{\pi}\left[2\sin x+\cos(2x)\right]dx=-2\cos x+\frac{1}{2}\sin 2x\Big|_0^{\pi}=\boxed{4}$

c) Numerical approximation $=4$ (see attached)

7.2 #10. $x=-y^2+4y$



Consider the disk at a y-point that is really slim:

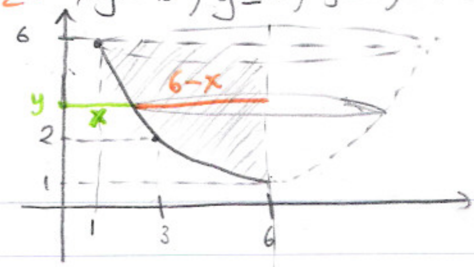
$r=x=-y^2+4y$

$\pi r^2dy=\pi(-y^2+4y)^2dy$

So Volume $=\int_{y=1}^{y=4}\pi(-y^2+4y)^2dy=\pi\int_1^4(-y^2+4y)^2dy$

$=\boxed{\frac{153\pi}{5}}$

7.2 #22. $xy=6,\ y=2,\ y=6,\ x=6$



$r=6-x$

$r=6-\frac{6}{y}$

Volume of disk $=\pi\left(6-\frac{6}{y}\right)^2dy$

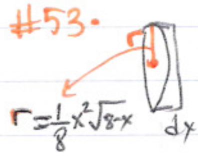
$\text{Volume}=\pi\int_1^6\left(6-\frac{6}{y}\right)^2dy$

$=\pi\int_1^6\left(36-\frac{72}{y}+\frac{36}{y^2}\right)dy$

$=36\pi\int_1^6\left(1-\frac{2}{y}+\frac{1}{y^2}\right)dy$

$=12\pi(13-6\ln 3)\approx 241.59$

#53. disk



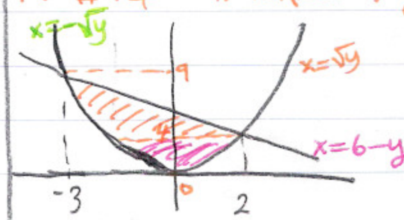
volume of disk $=\pi r^2dx$

$\text{Volume}=\pi\int_0^2\left(\frac{1}{8}x^2\sqrt{2-x}\right)^2dx$

$=\frac{1}{64}\pi\int_0^2x^4(2-x)\,dx$

$=\frac{\pi}{30}$.

7.1 #14 with respect to y.



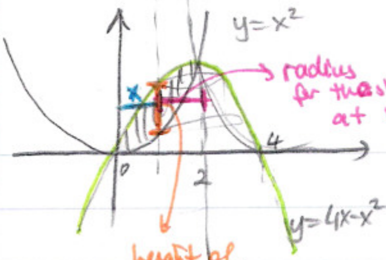
$\int_0^4(\sqrt{y}-(-\sqrt{y}))\,dy+$

$\int_4^9(6-y-(-\sqrt{y}))\,dy$

$=\int_0^4 2\sqrt{y}\,dy+\int_4^9(6-y+\sqrt{y})\,dy$

$=\frac{125}{6}$

7-3 #22 $y=x^2$, $y=4x-x^2$, about $x=2$



$4x-x^2=x^2$

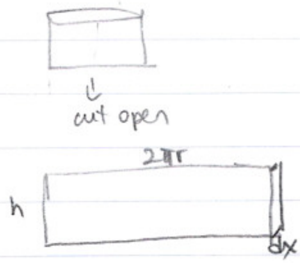
$\Rightarrow 2x^2-4x=0$

$2x(x-2)=0$

$x=0 \quad x=2$

$\| \; r=2-x$

$h=4x-x^2-x^2$



$$V=\int_0^2 2\pi r h$$

$$=2\pi\int_0^2 (2-x)(4x-2x^2)\,dx=\boxed{\frac{16\pi}{3}}$$

#26. $y=4-e^x$ about y-axis

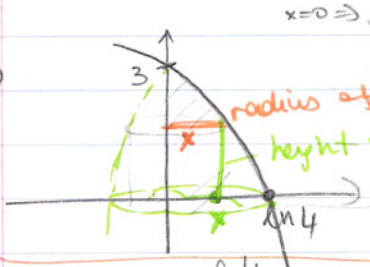
use shell method. DO NOT evaluate

$x=0 \Rightarrow y=4-e^0=3$

$4-e^x=0$

$\Rightarrow e^x=4$

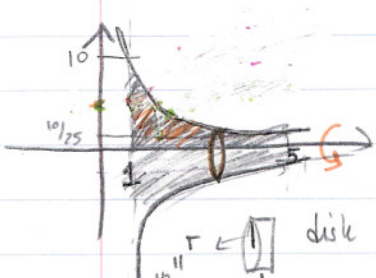
$x=\ln 4$



$$\boxed{\text{Volume}=\int_0^{\ln 4} 2\pi x(4-e^x)\,dx}$$

#28. $y=\frac{10}{x^2}$, $y=0$, $x=1$, $x=5$

a) rotate about the x-axis: (disk)

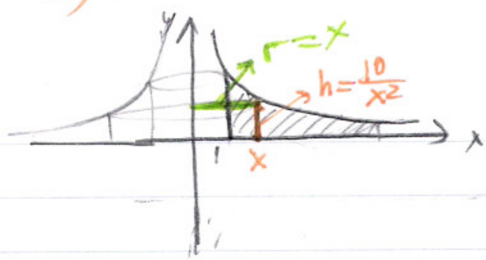


$$\text{Volume}=\pi\int_1^5\left(\frac{10}{x^2}\right)^2dx$$

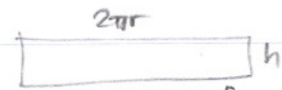
$$=\pi\int_1^5 100x^{-4}\,dx$$

$$=\frac{496}{15}\pi$$

28 b) rotate about the y-axis



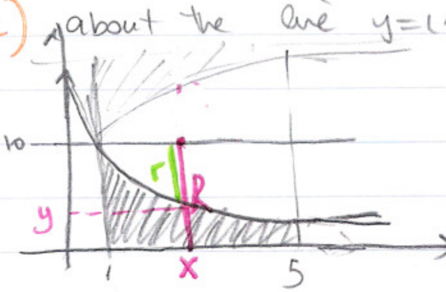
shell method: shells for an x-value



$$\text{Volume}=2\pi\int_1^5 x\cdot\frac{10}{x^2}\,dx=$$

$$=20\pi\ln 5$$

c) about the line $y=10$



disks (washers)

For an x-value, larger radius is R which is 10, smaller radius is r which is $10-y=10-\frac{10}{x^2}$

$$\text{So}\quad \text{Volume}=\pi\int(R^2-r^2)\,dx$$

$$=\pi\int_1^5 10^2-\left(10-\frac{10}{x^2}\right)^2dx$$

$$=\frac{1904}{15}\pi$$