MATH 150 HW 5 SOLUTIONS

Section 7-4 #4. $y = 2x^{3/2} + 3$

$y' = 3x^{1/2}$, $0 \le x \le 9$

arclength = $\int_0^9 \sqrt{1+(y')^2}\,dy = \int_0^9 \sqrt{1+9x}\,dx$ (from $x=0$ to $x=9$)

$= \frac{2}{27}(1+9x)^{3/2}\Big|_0^9 \approx \boxed{54.92}$

#20. $y = \cos x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$y' = -\sin x \Rightarrow 1+(y')^2 = 1+\sin^2 x$

length of arc $= L = \int_{-\pi/2}^{\pi/2} \sqrt{1+\sin^2 x}\,dx$

(see DERIVE file for graph and computation)

#30. $x^{2/3} + y^{2/3} = 4$

$y^{2/3} = 4 - x^{2/3}$

$y = (4-x^{2/3})^{3/2}$

$y' = \frac{3}{2}(4-x^{2/3})^{1/2}\left(-\frac{2}{3}x^{-1/3}\right)$

$= \frac{-(4-x^{2/3})^{1/2}}{x^{1/3}}$

$1+(y')^2 = 1 + \frac{4-x^{2/3}}{x^{2/3}} = \frac{4}{x^{2/3}}$

Length of the quarter of the arc is $= \int_0^8 \sqrt{\frac{4}{x^{2/3}}}\,dx$

$= \int_0^8 x^{-2/3}\,dx = 6x^{1/3}\Big|_0^8 = 12$

Then, length of arc $= 4 \times 12 = \boxed{48}$

38. $y = \sqrt{25-x^2}$

$y' = \frac{-x}{\sqrt{25-x^2}}$

$1+(y')^2 = 1 + \frac{x^2}{25-x^2} = \frac{25}{25-x^2}$

arc length $= \int_{-3}^{4} \sqrt{\frac{25}{25-x^2}}\,dx$

$= 5\arcsin\frac{x}{5}\Big|_{-3}^{4} =$

$5\left(\arcsin\frac{4}{5} - \arcsin\left(\frac{-3}{5}\right)\right)$

$\approx \boxed{7.8540}$ (Use DERIVE)

circumference of the circle $2\pi \cdot 5$

$\frac{1}{4}\cdot$ circumference $\approx \boxed{7.8540}$

40. $y = 2\sqrt{x}$

$y' = \frac{1}{\sqrt{x}} \Rightarrow 1+(y')^2 = 1+\frac{1}{x}$

Surface Area $= 2\pi\int_4^9 \underbrace{2\sqrt{x}}_{r}\ \underbrace{\sqrt{1+\frac{1}{x}}\,dx}_{dl\ (\text{arclength})}$

$= 4\pi\int_4^9 \sqrt{x+1}\,dx$

$= 4\pi\frac{2}{3}(x+1)^{3/2}\Big|_4^9$

$= \frac{8}{3}\pi\,(x+1)^{3/2}\Big|_{x=4}^{9}$

$= \frac{8}{3}\pi\left(10^{3/2} - 5^{3/2}\right) \approx 171.258$