Section 8.5 #10. $\int \frac{x+1}{x^2+4x+3}dx$

$$\frac{x+1}{x^2+4x+3} = \frac{A}{(x+3)} + \frac{B}{(x+1)}$$

get common denominator $\Rightarrow x+1 = A(x+1)+B(x+3)$

$\Rightarrow x+1 = (A+B)x + A+3B$

$\Rightarrow A+B=1$

$-(A+3B=1)$

$-2B=0$

$B=0 \Rightarrow A=1$

So $\int \frac{x+1}{x^2+4x+3}dx = \int \frac{1}{x+3}dx = \boxed{\ln|x+3|+C}$

#18. $\int \frac{2x-3}{(x-1)^2}dx$ $\quad \frac{2x-3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

get common denominator $\Rightarrow 2x-3 = A(x-1)+B$

$2x-3 = Ax-A+B$

$\Rightarrow A=2.$

$-A+B=-3 \Rightarrow -2+B=-3 \Rightarrow B=-1$

So, $\int \frac{2x-3}{(x-1)^2}dx = \int\left(\frac{2}{x-1} - \frac{1}{(x-1)^2}\right)dx$

$= \boxed{\ln|x-1| + \frac{1}{x-1} + C}$

#30. $\int_1^5 \frac{x-1}{x^2(x+1)}dx$ $\quad \frac{x-1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

get common denominator $\Rightarrow x-1 = A\,x(x+1) + B(x+1) + Cx^2$

$x-1 = Ax^2+Ax+Bx+B+Cx^2$

$\Rightarrow A+C=0,\quad A+B=1,\quad B=-1$

Then $A=2,\ C=-2$

So, $\int_1^5 \frac{x-1}{x^2(x+1)}dx = \int_1^5\left(\frac{2}{x} - \frac{1}{x^2} - \frac{2}{x+1}\right)dx$

$= 2\ln|x| + \frac{1}{x} - 2\ln|x+1|\Big|_1^5$

$= \left(2\ln 5 + \frac{1}{5} - 2\ln 6\right) - \left(2\ln 1 + \frac{1}{1} - 2\ln 2\right)$

$= -\frac{4}{5} + \ln\left(\frac{25\cdot 4}{36}\right) = \boxed{-\frac{4}{5} + \ln\left(\frac{100}{36}\right)}$

#40. $\int \frac{x^2-x+2}{x^3-x^2+x-1}dx$. point $(2,6)$

$$\frac{x^2-x+2}{x^3-x^2+x-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

get common denominator $\Rightarrow$

$x^2-x+2 = A(x^2+1)+(Bx+C)(x-1)$

$x^2-x+2 = Ax^2+A+Bx^2-Bx+Cx-C$

$\Rightarrow A+B=1$ (1), $-B+C=-1$ (2), $A-C=2$ (3).

(1) & (2) $\Rightarrow A+C=0$; (3) $A-C=2$ $\Rightarrow 2A=2 \Rightarrow A=1$

$A=1 \Rightarrow B=0,\ C=-1$

So, $\int \frac{x^2-x+2}{x^3-x^2+x-1}dx = \int\left(\frac{1}{x-1} - \frac{1}{x^2+1}\right)dx$

$= \ln|x-1| - \arctan(x) + C$

plug in $x=2$ and set equal to 6:

$\ln(2-1) - \arctan(2) + C = 6$

$\Rightarrow \boxed{C = 6+\arctan 2}$

Also see DERIVE file

#44. $\int \frac{\sec^2 x}{\tan x(\tan x+1)}dx$

Substitute $u=\tan x \Rightarrow du = \sec^2 x\,dx$

So the integral becomes $\int \frac{1}{u(u+1)}du$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

get common denominator

$\Rightarrow 1 = A(u+1)+Bu$

$1 = (A+B)u + A$

$\Rightarrow A=1,\quad A+B=0$

$1+B=0 \Rightarrow B=-1$

So $\int \frac{1}{u(u+1)}du = \int\left(\frac{1}{u} - \frac{1}{u+1}\right)du$

$= \ln|u| - \ln|u+1| + C$

$= \boxed{\ln|\tan x| - \ln|\tan x+1| + C}$