Section 9.4 #10. $0 < \frac{1}{\sqrt{n^3+1}} < \frac{1}{n^{3/2}}$

So $0 \le \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}} \le \underbrace{\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}}$

convergent by p-test
$p = \frac{3}{2} > 1$

So by Direct Comparison Test, $\sum \frac{1}{\sqrt{n^3+1}}$ is convergent.

#8. $0 < \frac{3^n}{4^n+5} < \left(\frac{3}{4}\right)^n$

So, $0 \le \sum_{n=0}^{\infty} \frac{3^n}{4^n+5} \le \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n$
$\Downarrow$
convergent by Geometric Series Test $r = \frac{3}{4} < 1$

So, $\sum_{n=0}^{\infty} \frac{3^n}{4^n+5}$ is convergent by Direct Comparison

#20. $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$ Dominant powers in numerator & denominator $\frac{n}{n^2} \to \frac{1}{n}$

Compare w.t $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\frac{5n-3}{n^2-2n+5}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{5n-3}{n^2-2n+5} \cdot \frac{n}{1} = 5 \text{ so,}$$

since $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent, $\sum \frac{5n-3}{n^2-2n+5}$ is divergent by Limit Comparison Test

#28. $\sum_{n=1}^{\infty} \tan 1/n$ compare with $\sum_{n=1}^{\infty} \frac{1}{n}$

$$\lim_{n\to\infty} \frac{\tan 1/n}{1/n} \to \frac{0}{0} \Rightarrow \text{L'Hospital} \quad \lim_{\cancel{x}\to\infty} \frac{\sec^2 \frac{1}{\cancel{x}} \cdot \left(-\frac{1}{\cancel{x^2}}\right)}{-\frac{1}{\cancel{x^2}}} = 1$$

Since $\sum \frac{1}{n}$ is divergent, by Limit comparison Test $\sum \tan \frac{1}{n}$ is also divergent.

Section 9.5 #14. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)} \to$ alternating with $a_n = \frac{1}{\ln(n+1)}$

- Since $\ln(n+2) > \ln(n+1)$
  we have $\frac{1}{\ln(n+2)} < \frac{1}{\ln(n+1)}$, that is $a_{n+1} < a_n$

- $\lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0$

By Alternating Series Test $\sum \frac{(-1)^n}{\ln(n+1)}$ is convergent

Section 9.5 #26. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \to$ Alternating with $a_n = \frac{1}{(2n+1)!}$

- Since $\frac{1}{(2(n+1)+1)!} \overset{?}{<} \frac{1}{(2n+1)!}$

This is true because $(2n+3)! > (2n+1)!$

So $a_{n+1} < a_n$.

- $\lim_{n\to\infty} \frac{1}{(2n+1)!} = 0$

So by Alternating Series Test $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$ is convergent

#50 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ Check for absolute convergence:

$$\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{n\sqrt{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \to \text{convergent by p-test } (p = \tfrac{3}{2} > 1)$$

Since $\sum \frac{(-1)^{n+1}}{n\sqrt{n}}$ is absolutely convergent, it is convergent itself.

#52. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(2n+3)}{n+10}$ $\quad \lim_{n\to\infty} (-1)^{n+1} \frac{(2n+3)}{n+10}$ $\to 2$ for odd n, $\to -2$ for even n

So, not zero!

By nth term test, $\sum \frac{(-1)^{n+1}(2n+3)}{n+10}$ is divergent.

#59. $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ (Note that $\cos(0\pi) = 1$, $\cos(\pi) = -1$, $\cos(2\pi) = 1$, $\cos(3\pi) = -1$, $\vdots$)

Check for absolute convergence

$$\sum_{n=0}^{\infty} \left|\frac{(-1)^n}{n+1}\right| = \sum_{n=0}^{\infty} \frac{1}{n+1} \to \text{divergent (compare with } \sum \tfrac{1}{n} \text{ using Limit Comparison Test)}$$

So, $\sum \frac{(-1)^n}{n+1}$ is not absolutely convergent.

Is it conditionally convergent. Yes, we can use alternating series test: $\sum \frac{(-1)^n}{n+1}$ and $a_n = \frac{1}{n+1}$ $\to$ decreasing because $\frac{1}{n+2} < \frac{1}{n+1}$

$\lim_{n\to\infty} \frac{1}{n+1} = 0$