MATH 360 HOMEWORK 0 SOLUTIONS

1) Prove that there are infinitely many primes.

Suppose there are finitely many primes and $p_1, p_2, \dots, p_n$ are all of them.

Let $m = p_1 \cdot p_2 \cdots p_n + 1$.

If $p_i \mid m$ then $p_i \mid m - p_1 \cdot p_2 \cdots p_n$ (b/c $p_i \mid p_1 p_2 \cdots p_n$)

That is $p_i \mid 1$ which cannot happen because $p_i$ is prime hence is greater than 1.

This argument is true for any $i = 1, 2, \dots, n$.

So $m$ doesn't have any (prime) divisors except 1 and itself.

That is $m$ is prime.

But $p_1, p_2, \dots, p_n$ are ~~the~~ all the primes according to our assumption in the beginning.

This gives a contradiction with the assumption that there are finitely many primes.

So, there are infinitely many primes.

2) Prove that $1 + 2 + \dots + n = \frac{n(n+1)}{2}$ for every $n \in \mathbb{Z}^+$.

Let $1 + 2 + \dots (n-1) + n = S$

Then $n + (n-1) + \dots + 2 + 1 = S$ as well.

Add the two

$$\underbrace{(n+1) + (n+1) + \dots + (n+1) + (n+1)}_{n \text{ of them}} = 2S.$$

So, $n(n+1) = 2S \Rightarrow \boxed{S = \frac{n(n+1)}{2}}$